http://www.th.phys.titech.ac.jp/~muto/lectures/NPI05/NPI05_chap03.ppt

原子核が存在するとは

- 寿命がある程度長くないと，存在が確認できない
  ある程度とはどのくらいか？

○ 光が原子核を通過するのに要する時間程度：
原子核の大きさを L = 10 fm = 10^{-14} m として

$$\frac{10^{-14}\ [\mathrm{m}]}{3\times 10^{8}\ [\mathrm{m}\cdot\mathrm{s}^{-1}]} \approx 3\times 10^{-23}\ \mathrm{s}$$

○ 強い相互作用による直接反応（direct reaction）に要する時間

$$\approx\ 10^{-22}\ \mathrm{s}$$

○ 速いγ遷移をする励起状態の寿命

$$\approx\ 10^{-15}\ \mathrm{s}$$

○ 原子番号が 113 の原子核（理研で確認）の寿命

$$^{70}\mathrm{Zn} + {}^{209}\mathrm{Bi} \rightarrow {}^{279}113 \rightarrow {}^{278}113 + n$$

$$T_{1/2}({}^{278}113)\ =\ 0.344\ \mathrm{ms}$$

素核宇宙融合　レクチャーシリーズ

第4回「原子核殻模型の基礎と応用」

阿部　喬（東大CNS）
京大基研

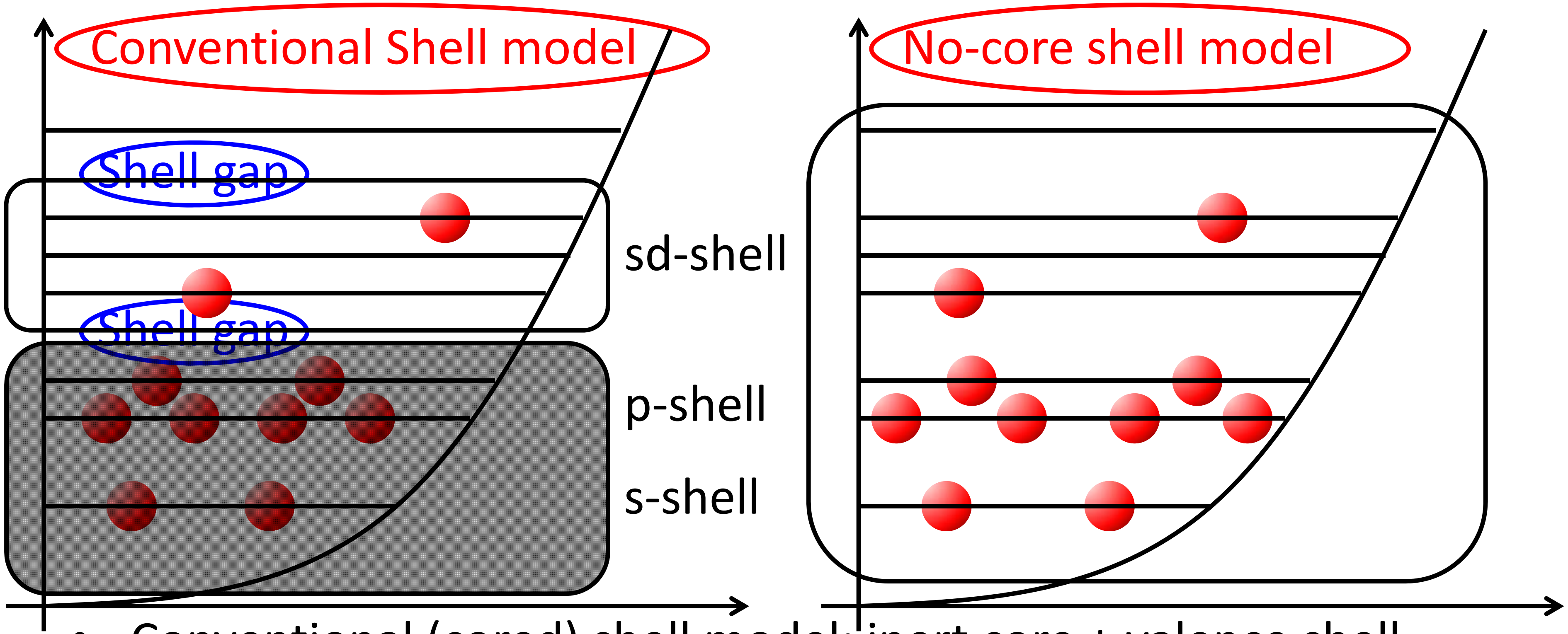
2012年1月11,12日

目次

- 核子の一粒子運動と原子核での殻構造
- 閉殻を仮定する（芯のある）殻模型計算の基礎
- 閉殻を仮定しない（芯のない）殻模型による
  第一原理計算の概要
- モンテカルロ殻模型

Image of the no-core shell model

- Conventional (cored) shell model vs. No-core shell model (NCSM)



- Conventional (cored) shell model: inert core + valence shell
- No-core shell model: All nucleons are active

No-core shell model

- A large sparse matrix eigenvalue problem

$$H = T_{rel} + V_{NN} + V_{NNN} + \cdots$$

$$H|\Psi\rangle = E|\Psi\rangle \qquad |\Psi\rangle = \sum_n A_n|\Phi_n\rangle$$

Diagonalize $\{\langle\Phi_m|H|\Phi_n\rangle\}$

- Adopt realistic NN (& NNN) interaction(s) renormalization as needed – retain included many-body interactions: Chiral EFT & JISP16 interaction
- Adopt the 3-D HO for the single-particle basis states
- Evaluate the nuclear Hamiltonian in basis space of HO SDs
- Diagonalize this sparse many-body H in its “m-scheme” basis

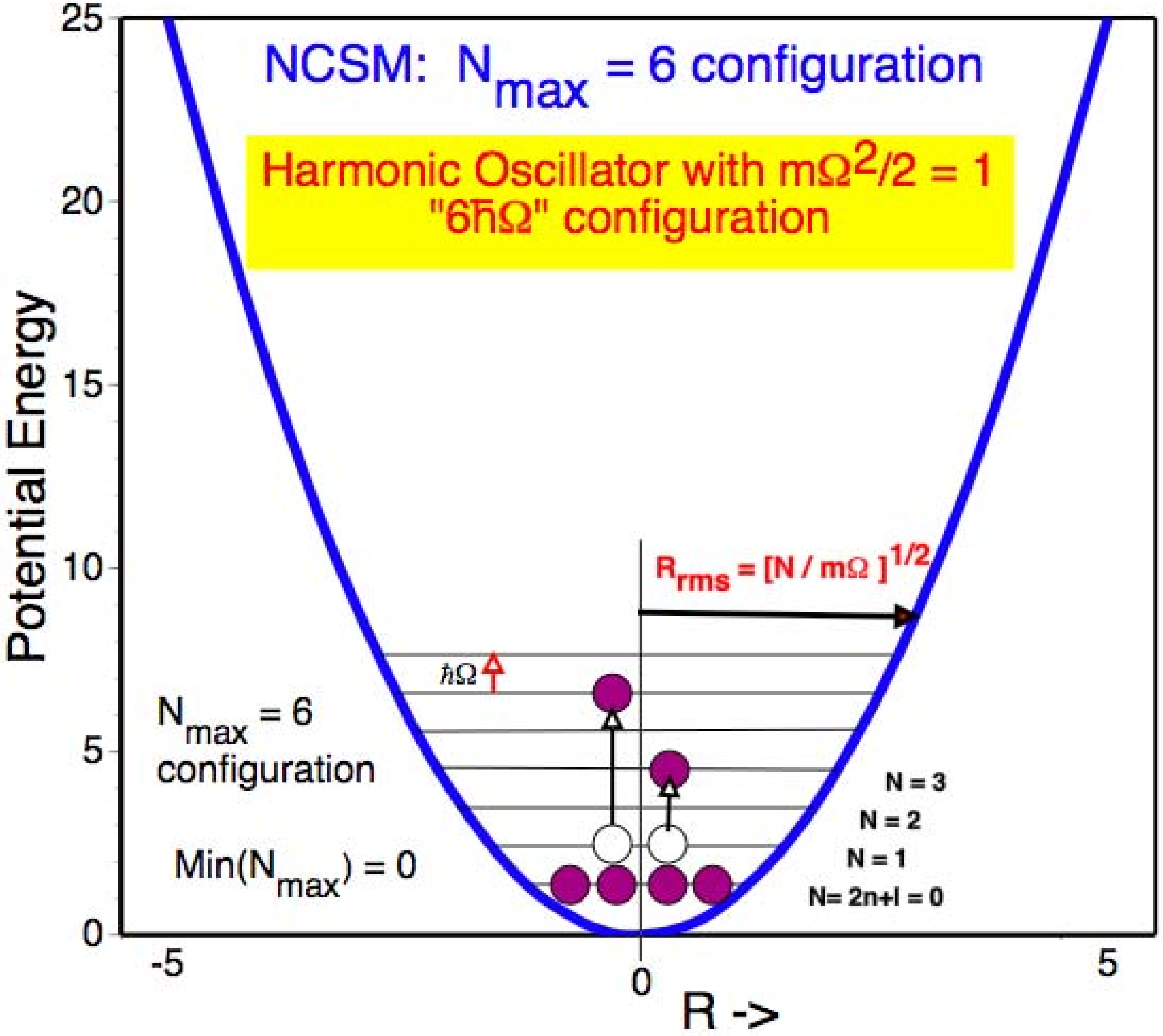
$$|\Phi_n\rangle = \left[a^\dagger_\alpha a^\dagger_\beta \cdots a^\dagger_\zeta\right]_n |0\rangle \qquad n = 1,2, \ldots, 10^{10} \text{ or more…}$$

- Evaluate obs & compare w/ exp.

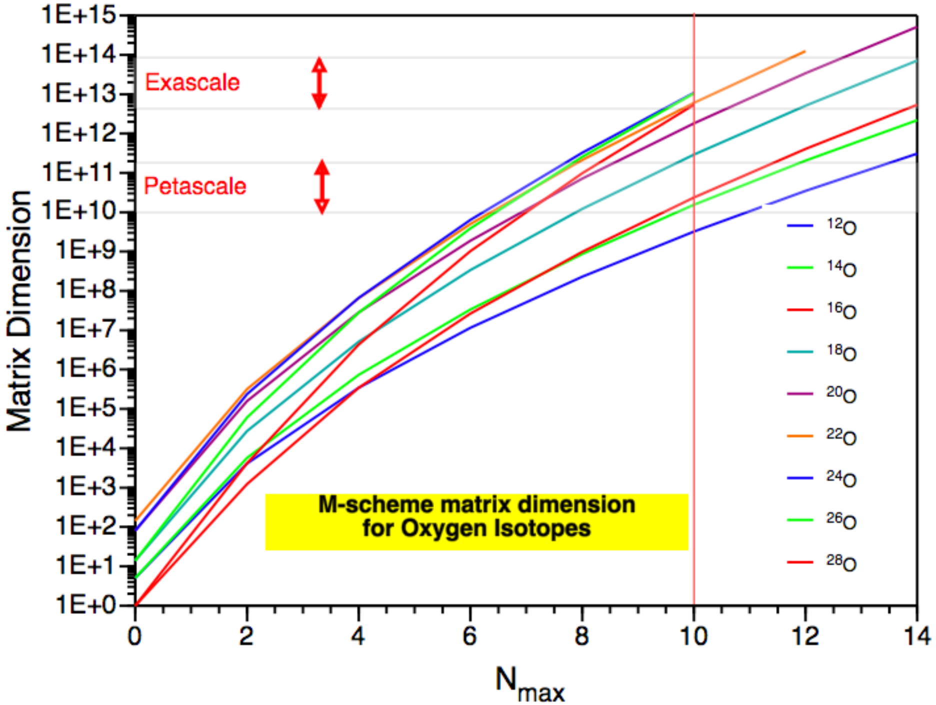
Some comments

- Straightforward but computationally demanding
  -> new algorithms/computers
- Require convergence assessments & extrapolation tools
- Achievable for nuclei up to A = 16 (40) today w/ largest computers available

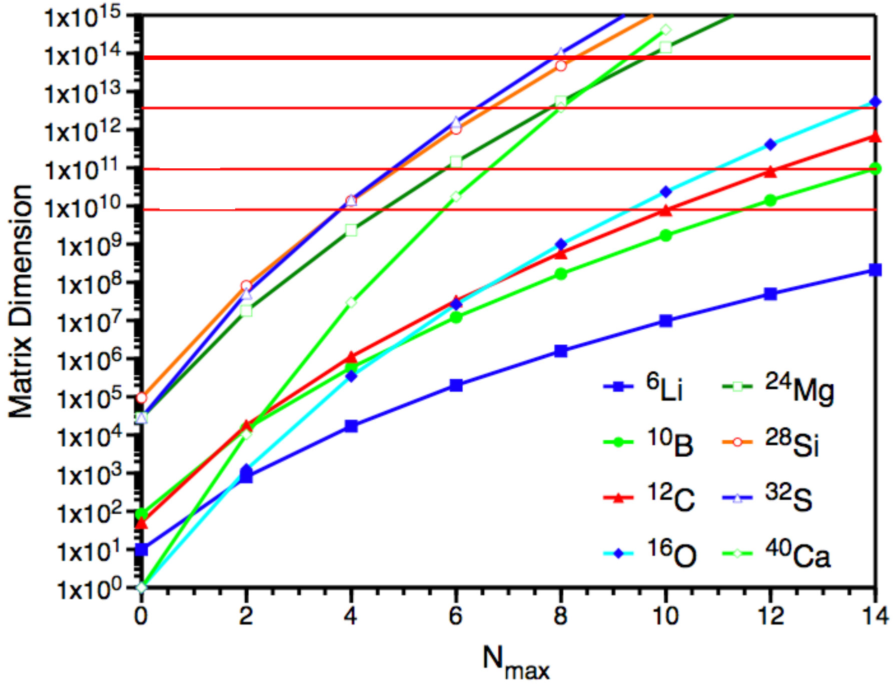
- No-core shell model (NCSM):
  effective interaction in the Nmax truncation
- No-core full configuration (NCFC):
  bare interaction in the Nmax truncation w/ the model-space extrp. (Nmax -> infinite)



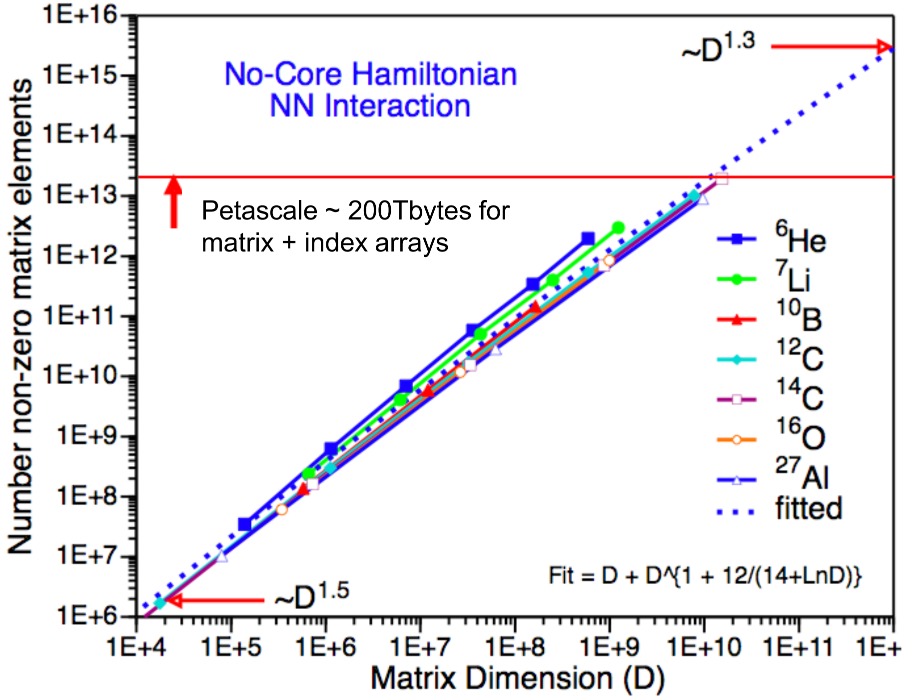
NCSM: N max = 6 configuration
Harmonic Oscillator with mΩ2/2 = 1
"6ħΩ" configuration
Potential Energy
25
20
15
10
5
0
-5
0
5
R ->
Rrms = [N / mΩ]1/2
ħΩ
Nmax = 6
configuration
Min(Nmax) = 0
N = 3
N = 2
N = 1
N= 2n+l = 0



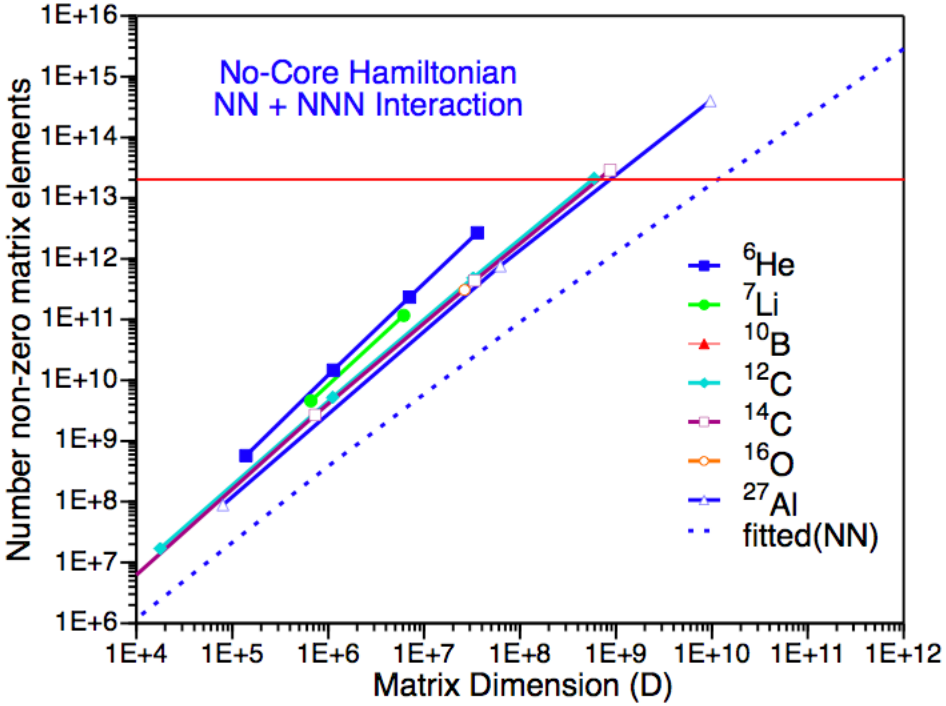
Exascale
Petascale
M-scheme matrix dimension
for Oxygen Isotopes
Matrix Dimension
N_max
12O
14O
16O
18O
20O
22O
24O
26O
28O



Matrix Dimension
N_max
6Li
10B
12C
16O
24Mg
28Si
32S
40Ca



No-Core Hamiltonian
NN Interaction
~D^1.3
Petascale ~ 200Tbytes for
matrix + index arrays
~D^1.5
Fit = D + D^{1 + 12/(14+LnD)}
Number non-zero matrix elements
Matrix Dimension (D)
6He
7Li
10B
12C
14C
16O
27Al
fitted



No-Core Hamiltonian
NN + NNN Interaction
Number non-zero matrix elements
Matrix Dimension (D)
6He
7Li
10B
12C
14C
16O
27Al
fitted(NN)

ab inito NCSM

- Effective Hamiltonian for A-particles

  (Lee-Suzuki-Okamoto method + Cluster decomposition)

P. Navratil, J.P. Vary and B.R. Barrett,
Phys. Rev. Lett. 84, 5728(2000); Phys. Rev. C62, 054311(2000)
C. Viazminsky and J.P. Vary, J. Math. Phys. 42, 2055 (2001);
K. Suzuki and S.Y. Lee, Progr. Theor. Phys. 64, 2091(1980);
K. Suzuki, *ibid*, 68, 246(1982);
K. Suzuki and R. Okamoto, *ibid*, 70, 439(1983)

Preserves the symmetries of the full Hamiltonian:
Rotational, translational, parity, etc., invariance

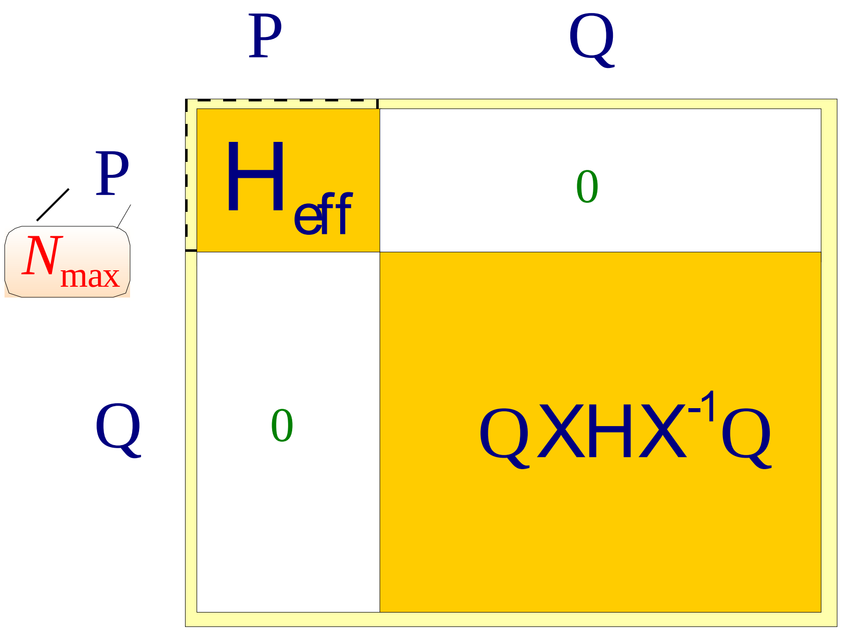
$$H_{\mathrm{A}} = T_{rel} + V = \sum_{i<j}^{\mathrm{A}} [\frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + V_{ij}] + V_{NNN}$$

Select a finite oscillator basis space (P-space) and evaluate an
 - body cluster effective Hamiltonian:

$$H_{eff} = P\left[T_{rel} + V^a(N_{\max}, \hbar\Omega)\right]P$$

Guaranteed to provide exact answers as $a \to A$ or as $P \to 1$.

Effective Hamiltonian in the NCSM
Lee-Suzuki-Okamoto renormalization scheme



$$H: \quad E_1, E_2, E_3, \ldots E_{d_P}, \ldots E_\infty$$

$$H_{eff}: \quad E_1, E_2, E_3, \ldots E_{d_P}$$

model space dimension

$$QXHX^{-1}P = 0$$

$$H_{eff} = PXHX^{-1}P$$

unitary $X = \exp[-\operatorname{arctan h}(\omega^+ - \omega)]$

- n-body cluster approximation, $2 \leq n \leq A$
- $H^{(n)}_{eff}$ n-body operator
- Two ways of convergence:
  - For $P \rightarrow 1$ $H^{(n)}_{eff} \rightarrow H$
  - For $n \rightarrow A$ and fixed P: $H^{(n)}_{eff} \rightarrow H_{eff}$

Some NCSM Results

Current Status of No-Core Shell Model (NCSM)

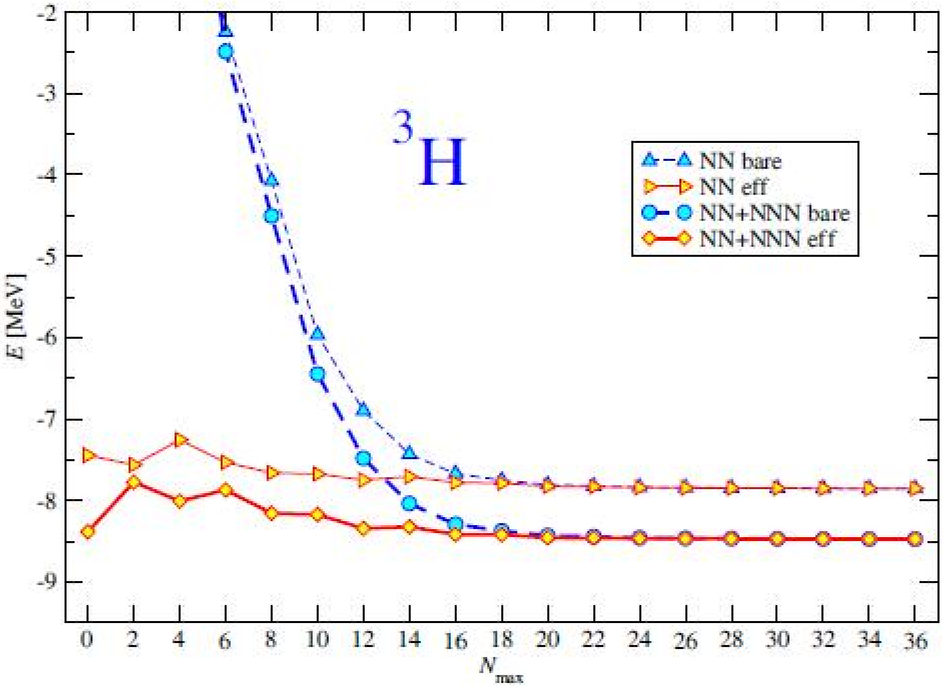


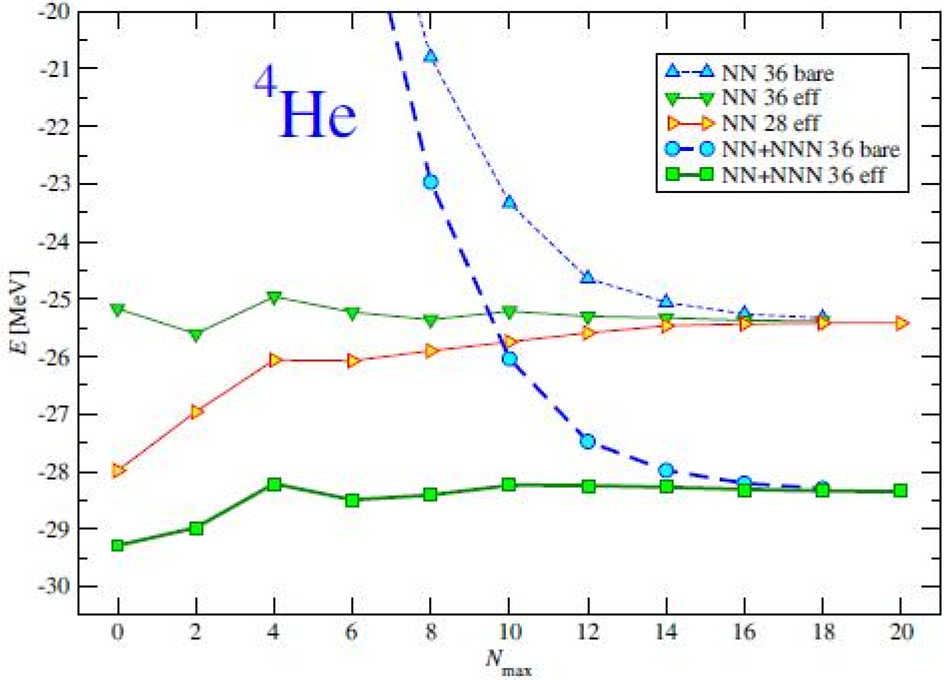
Fig. 1. – ^{3}H ground-state energy dependence on the size of the basis. The HO frequency of $\hbar\Omega = 28$ MeV was employed. Results with (thick lines) and without (thin lines) the NNN interaction are shown. The full lines correspond to calculations with two-body effective interaction derived from the chiral NN interaction, the dashed lines to calculations with the bare chiral NN interaction.

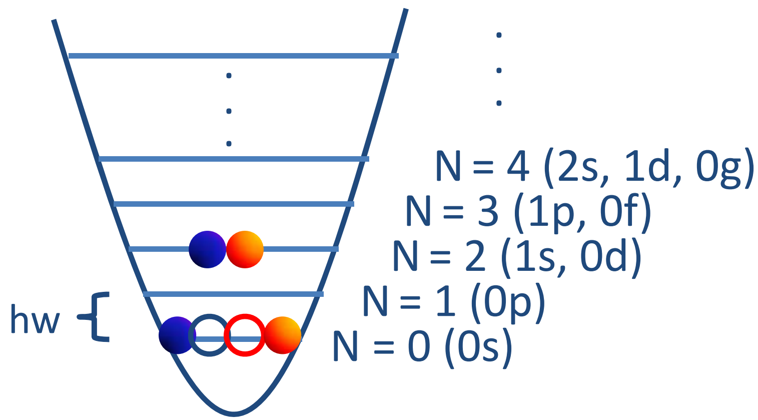
P. Navratil, Enrico Fermi Lecture (2007)

Current Status of No-Core Shell Model (NCSM)

- Nmax-truncation (NCSM, NCFC)
  - ➢ Max. # of HO quanta of many-body basis

<u>Nmax = 4</u> (A = 4)





$$N = \sum_i 2n_i + l_i \leq N_{max}$$

Fig. 2. – ^{4}He ground-state energy dependence on the size of the basis. The HO frequencies of $\hbar\Omega = 28$ and 36 MeV was employed. Results with (thick lines) and without (thin lines) the NNN interaction are shown. The full lines correspond to calculations with three-body effective interaction, the dashed lines to calculations with the bare interaction. For further details see the text.

P. Navratil, Enrico Fermi Lecture (2007)

Current Status of No-Core Shell Model (NCSM)

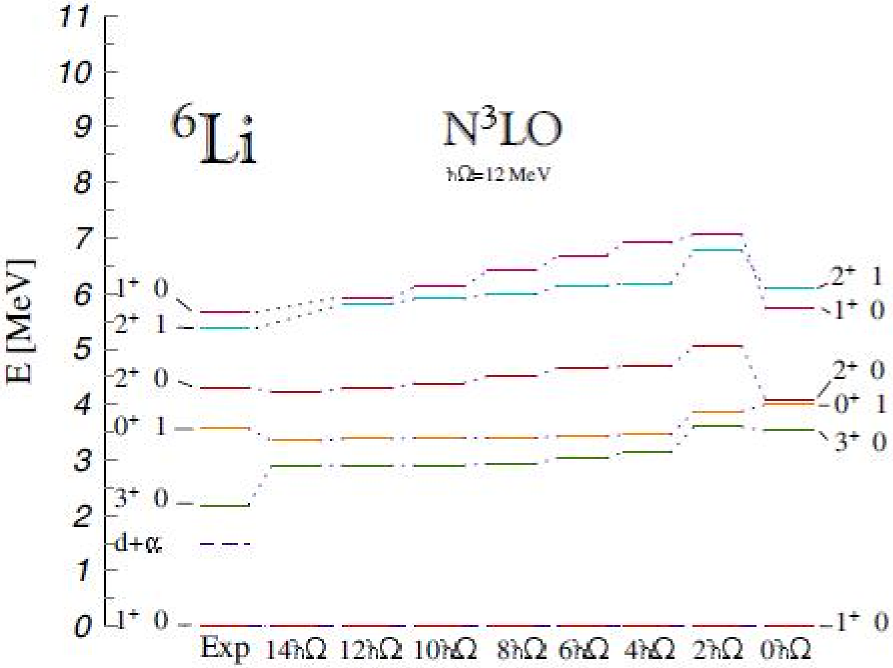


Fig. 3. – Calculated positive-parity excitation spectra of ^{6}Li obtained in $0\hbar\Omega$-$14\hbar\Omega$ basis spaces using two-body effective interactions derived from the chiral EFT NN potential are compared to experiment. The HO frequency of $\hbar\Omega = 12\,\text{MeV}$ was used.

P. Navratil, Enrico Fermi Lecture (2007)

Current Status of No-Core Shell Model (NCSM)

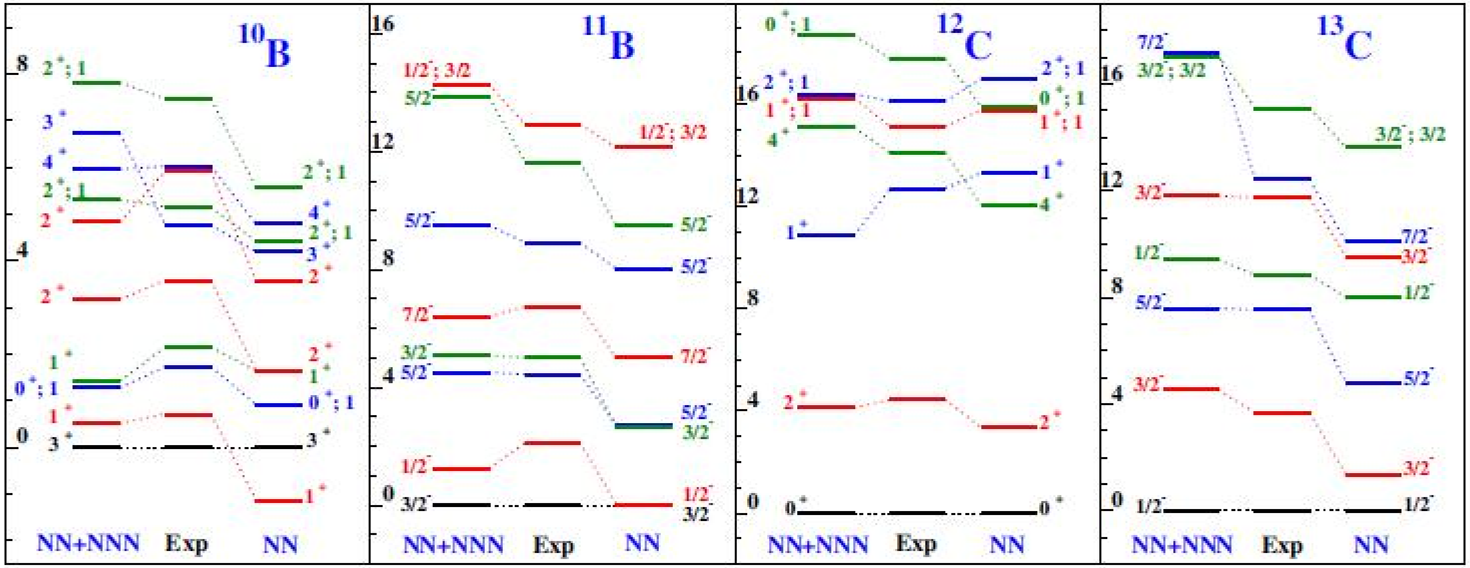


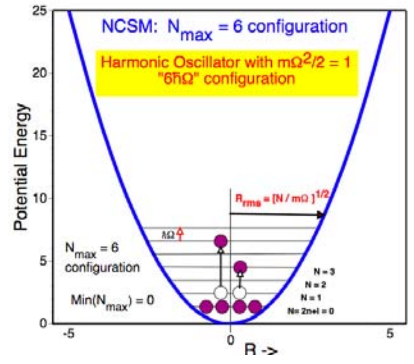
Fig. 10. – States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin $T = 0$ or $1/2$, the isospin label is explicitly shown only for states with $T = 1$ or $3/2$. The excitation energy scales are in MeV.

P. Navratil, Enrico Fermi Lecture (2007)

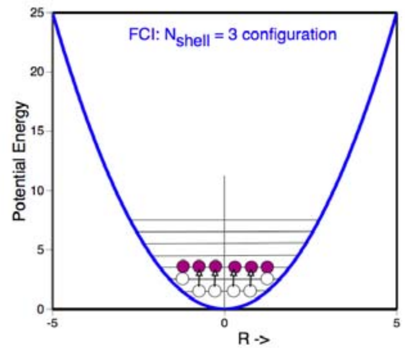
Slide from J.P. Vary

Work in progress: Different truncation schemes

- No-Core Shell Model, No-Core Full Configuration
  - truncation on total number of H.O. quanta, N_{max}, in many-body basis space



- Full Configuration Interaction
  - truncation on single-particle basis space, retaining all many-body states allowed by the symmetries



- Monte-Carlo Shell Model
  Abe, Otsuka, Shimizu, Utsuno
  - sampling of the many-body basis in a FCI truncation
- Importance Sampling
  Navratil, Roth
  - sampling of the many-body basis in a N_{max} truncation
- Symplectic No-Core Shell Model — Draayer *et.al.*, PetaApps grant

END