素核宇宙融合　レクチャーシリーズ

第4回「原子核殻模型の基礎と応用」

阿部　喬（東大CNS）
京大基研

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目次

- 核子の一粒子運動と原子核での殻構造
- 閉殻を仮定する（芯のある）殻模型計算の基礎
- 閉殻を仮定しない（芯のない）殻模型による第一原理計算の概要
- モンテカルロ殻模型

Summary of 1st lecture

- Nucleon: Single-particle motion

- Independent Particle Model

➢ HO (central) potential + LS splitting

-> Mayer-Jensen's magic #

Bohr -Mottelson

Single-particle orbits

WS (HO) central potential + spin-orbit interaction

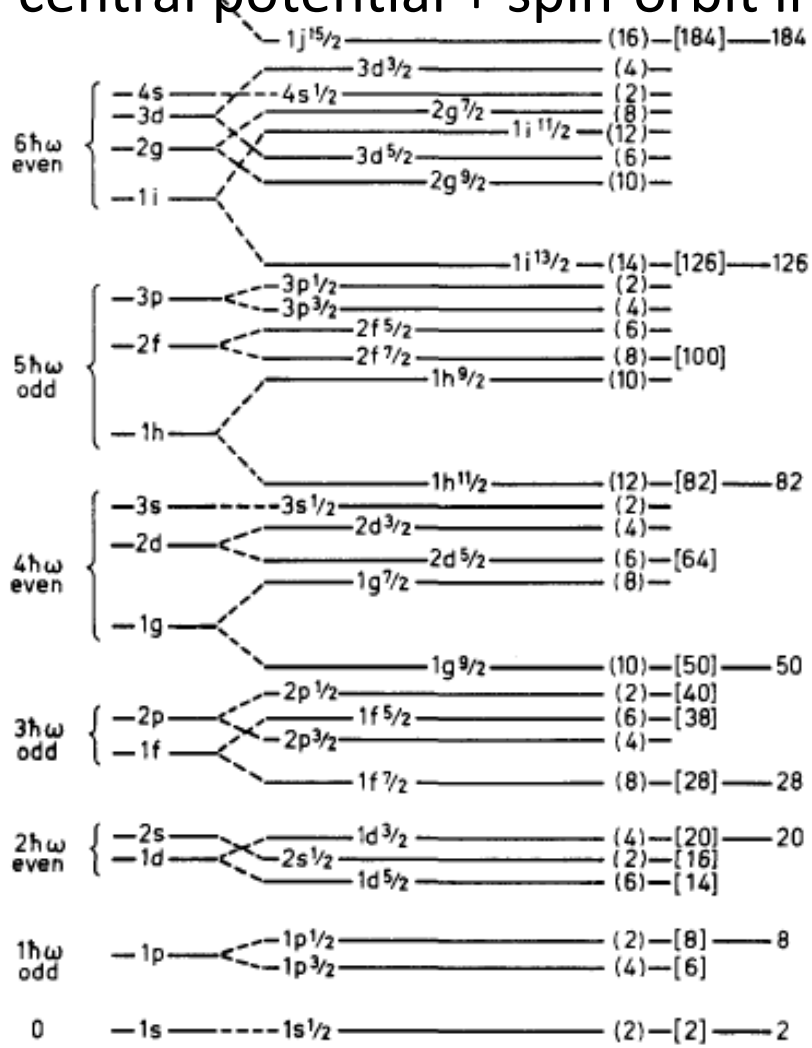
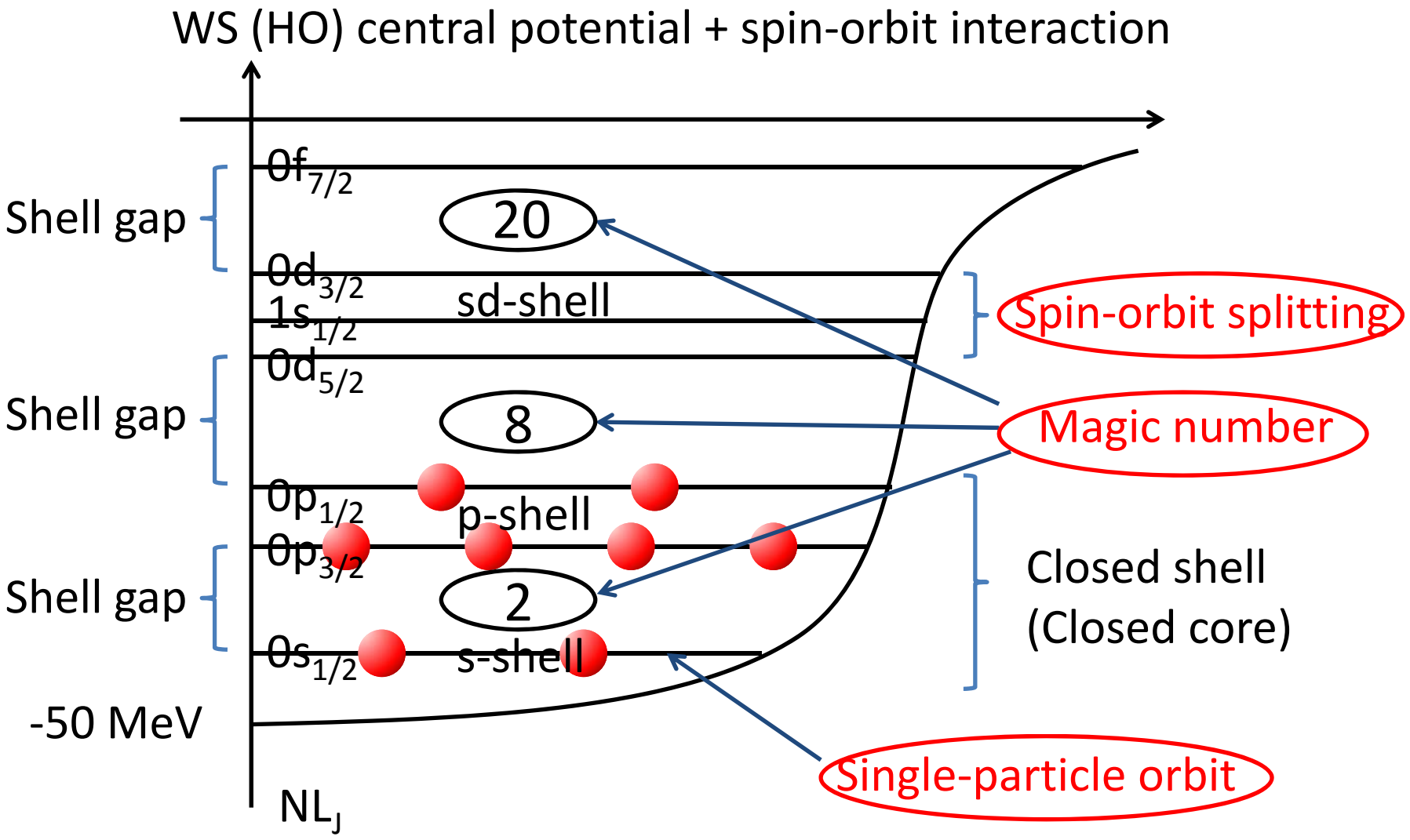


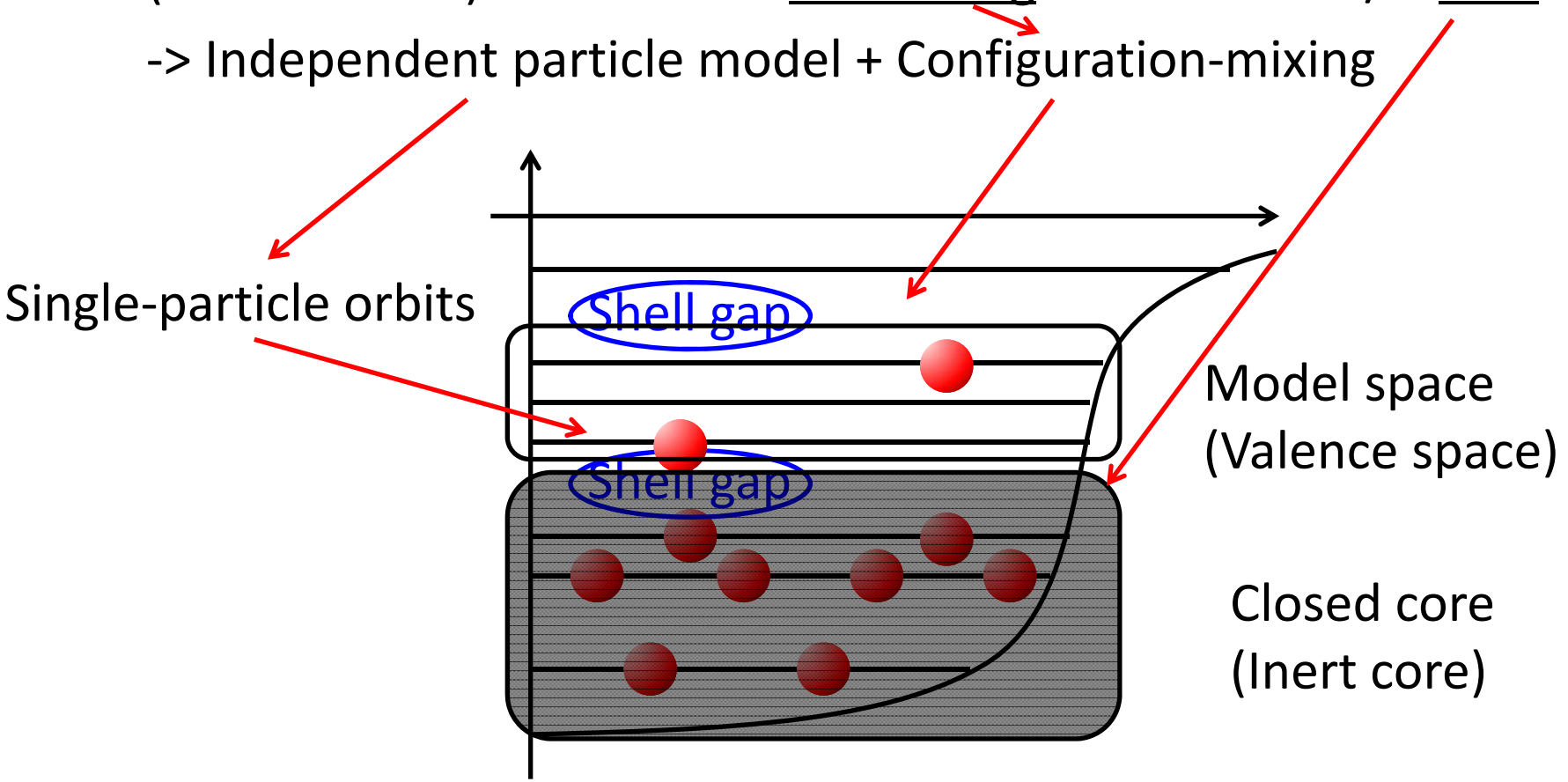
Figure 2-23 Sequence of one-particle orbits. The figure is taken from M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

Schematic picture of the single-particle potential



Conventional shell model

- (Conventional) Shell model: Interacting shell model w/ a core
  -> Independent particle model + Configuration-mixing



Shell-Model Hamiltonian

$$H = \sum_{i=1}^{N_{sps}} \epsilon_i n_i + \sum_{ijkl}^{N_{sps}} v_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$$

$$n_i = a_i^\dagger a_i$$

ϵ_i : Single-particle energy

v_{ijkl} : Two-body matrix element (TBME)

- w/ some assumptions/approximations

Image of the shell-model Hamiltonian

- Hamiltonian:

$$H = \sum_{i=1}^{A} T_i + \frac{1}{2}\sum_{i\neq j}^{A} V_{ij}$$

- Shell-model Hamiltonian:

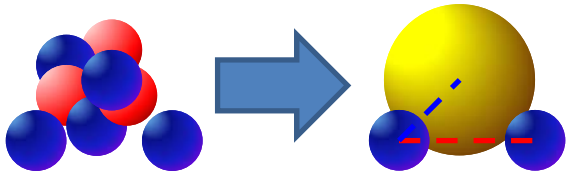
$$\begin{aligned} H &= \sum_{i=1}^{A}(T_i + U_i) + \frac{1}{2}\sum_{i\neq j}^{A} V_{ij} - \sum_{i=1}^{A} U_i \\ &= H_0 + H_1 \end{aligned}$$

$$H_0 = \sum_{i=1}^{A}(T_i + U_i) \qquad H_1 = \frac{1}{2}\sum_{i\neq j}^{A} V_{ij} - \sum_{i=1}^{A} U_i$$

One-body (mean) field (一体場)　　Residual interaction (残留相互作用 ; 摂動項)

Shell-model Hamiltonian

- Shell-model Hamiltonian



$$
\begin{aligned}
H &= \sum_{i=1}^{A} T_i + \frac{1}{2}\sum_{i\neq j}^{A} V_{ij} \\
&= \sum_{i\leq A_c} T_i + \sum_{i<j\leq A_c} V_{ij} + \sum_{A_c<i} T_i + \sum_{i\leq A_c, A_c<j} V_{ij} + \sum_{A_c<i<j} V_{ij} \\
&= H_c + H_v
\end{aligned}
$$

$$
H_c = \sum_{i\leq A_c} T_i + \sum_{i<j\leq A_c} V_{ij}
$$

$$
H_0 = \sum_{A_c<i} (T_i + U_i) \quad U_j = \sum_{i\leq A_c} V_{ij}
$$

$$
H_v = H_0 + V_v
$$

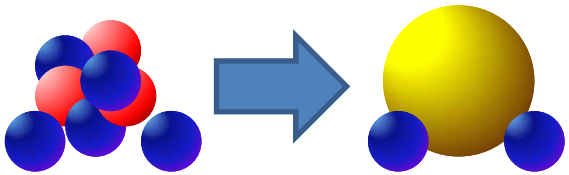
$$
V_v = \sum_{A_c<i<j} V_{ij}
$$

$H = H_0 + H_1$ (previous slide)

Shell-Model Hamiltonian

$$H = H_0 + H_1$$

$$H = \sum_{i=1}^{N_{sps}} \epsilon_i n_i + \sum_{ijkl}^{N_{sps}} v_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$$



$$n_i = a_i^\dagger a_i$$

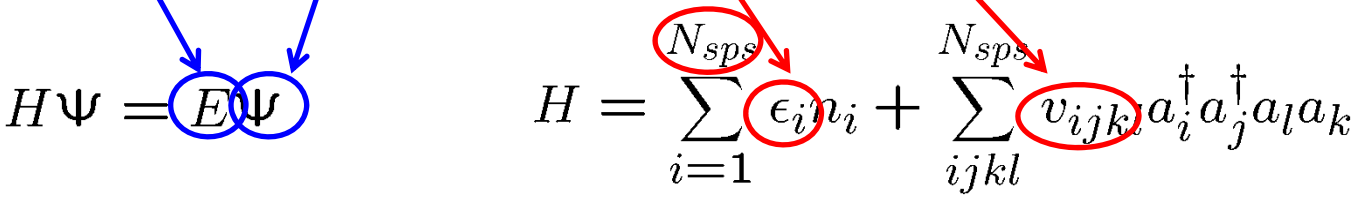
ϵ_i : Single-particle energy

v_{ijkl} : Two-body matrix element (TBME)

- w/ some assumptions/approximations

Procedure

1. Set the model space (valence space)
2. Get the single-particle energies (SPEs) from the exp. values
3. Compute the two-body matrix elements (TBMEs)
4. Diagonalize the Hamiltonian matrix (to get the energy eigenvalues & eigenfunctions) -> Configuration mixing
5. Calculate the other obs. (w/ the w.f. obtained above)



Model space (single-particle states)

Bohr -Mottelson

Single-particle orbits

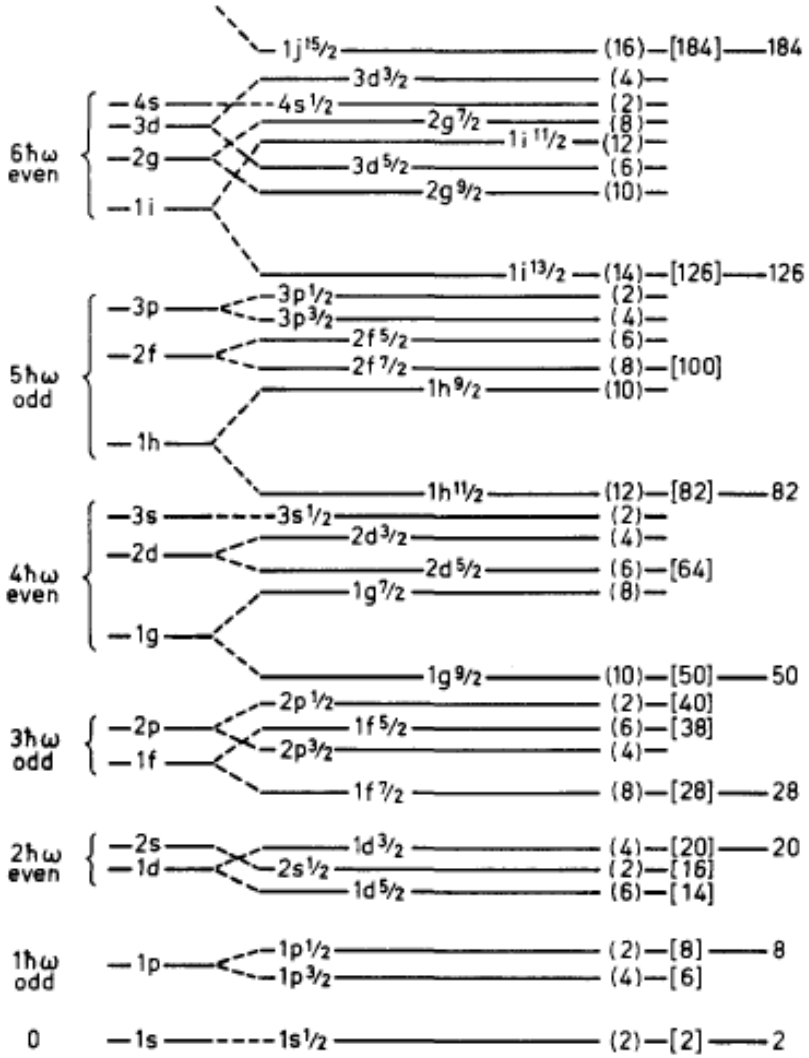


Figure 2-23 Sequence of one-particle orbits. The figure is taken from M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

NL_J

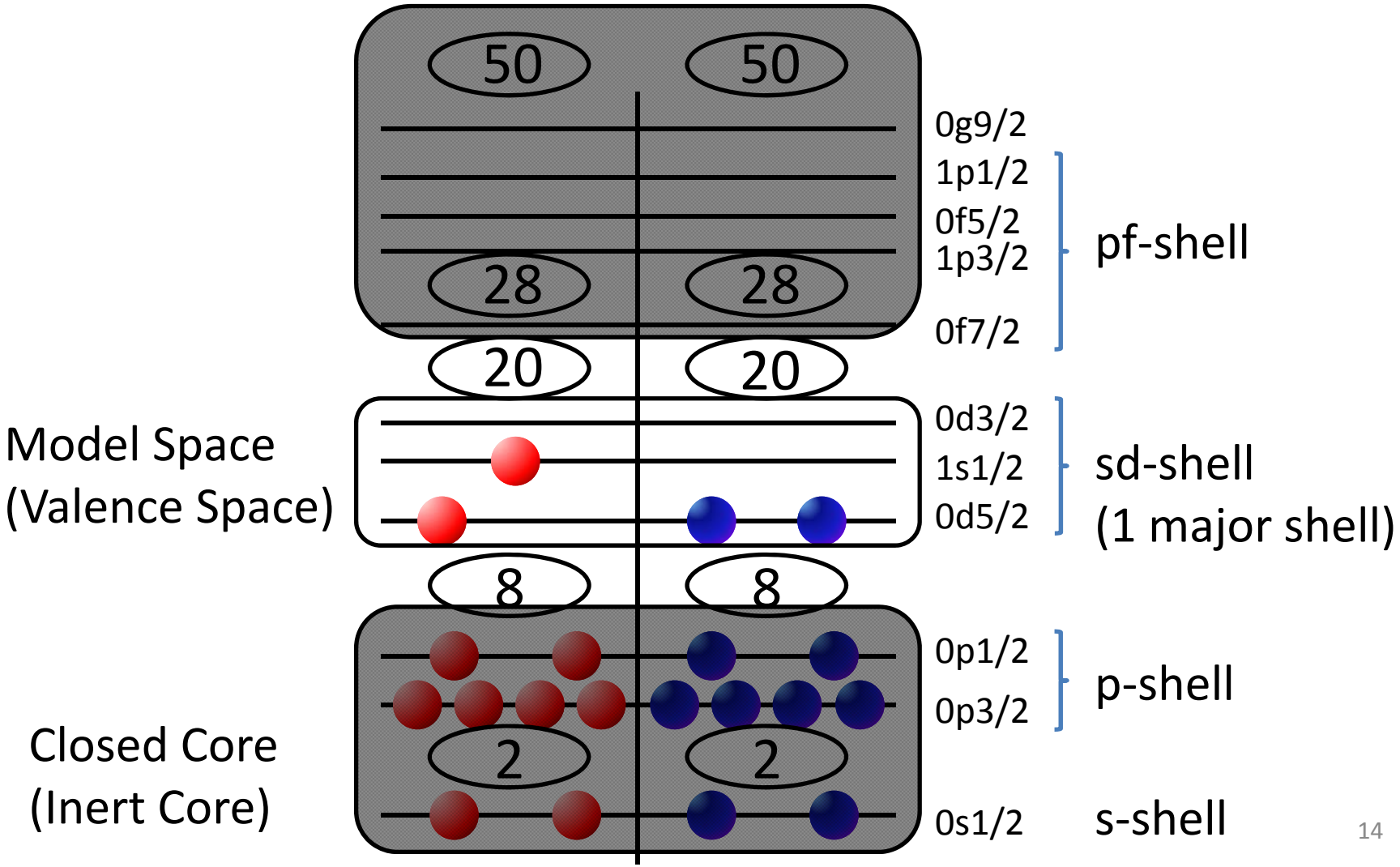
$N = 2n + l\ (+ 1)$

$= 1, 2, 3, \ldots$ $\quad J = l + s$

$(= 0, 1, 2, \ldots)$

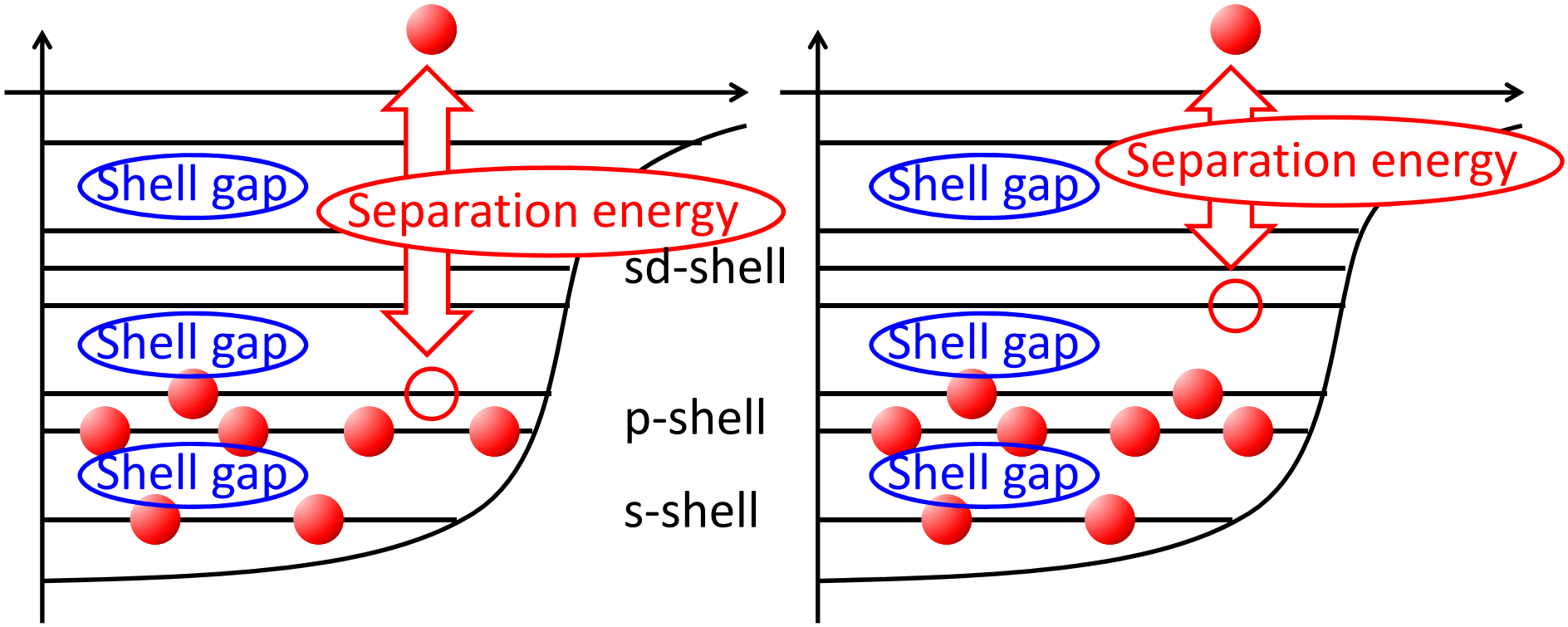
Orbital Angular Momentum	Symbol
l = 0	s
l = 1	p
l = 2	d
l = 3	f
l = 4	g
l = 5	h
l = 6	i

Model space for conventional shell model



Separation energy

- Separation energy: minimum energy to take a neutron/proton out

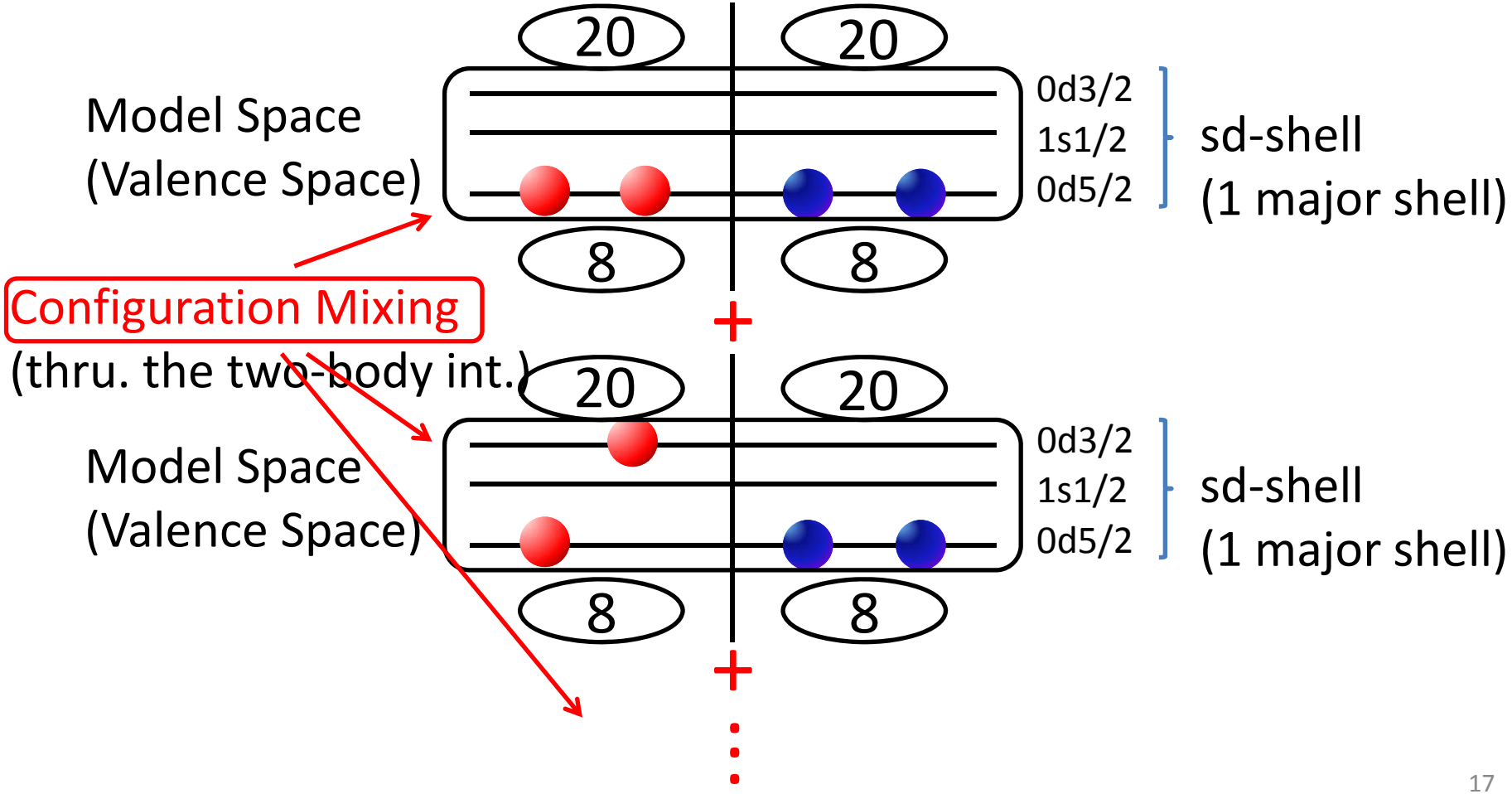


- Decrease suddenly after crossing the shell gap

Configuration mixing

Configuration mixing

- Configuration: pattern of occupation



Many-body wave function

- Single-particle Hamiltonian:

$$H_0 = \sum_{i=1}^{A} (T_i + U_i)$$

- Eigenvalue Equation:

$$H_0 \psi_i = \epsilon_i \psi_i$$

Single-particle w.f. Single-particle energy

- Many-body wave function:

$$\Psi_A \rightarrow \sum_{i}^{N_{sps}} x_i \mathcal{A} \left[\psi_1 \psi_2 \cdots \psi_A \right]$$

- Correlations are included by the configuration mixing.

Basis state

- Basis state (function):

$$\Phi = \frac{1}{\sqrt{n!}} \underbrace{\begin{vmatrix} \psi_a(\mathbf{r}_1) & \psi_a(\mathbf{r}_2) & \cdots & \psi_a(\mathbf{r}_{A_v}) \\ \psi_b(\mathbf{r}_1) & \psi_b(\mathbf{r}_2) & \cdots & \psi_b(\mathbf{r}_{A_v}) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_k(\mathbf{r}_1) & \psi_k(\mathbf{r}_2) & \cdots & \psi_k(\mathbf{r}_{A_v}) \end{vmatrix}}_{A_v} \Bigg\} A_v$$

$$A_v = A - A_c$$

$$J \geq M$$

Combination: $\begin{pmatrix} N_{sps} \\ A_v \end{pmatrix}$

一粒子波動関数の積であるSlater 行列式はJをよい量子数としてもたない（異なるJの成分が混ざる）

Conservation of M: $M = m_1 + m_2 + \cdots + m_k$

Possible configuration: $\begin{pmatrix} N_{sps} \\ A_v \end{pmatrix}$ の中でMを満たすもの: N_b

Eigenvalue Equation

- Many-body wave function:

$$\Psi = \sum_{k=1}^{N_b} x_k \Phi_k = x_1\Phi_1 + x_2\Phi_2 + \cdots + x_{N_b}\Phi_{N_b} \quad N_b : \#(\text{basis})$$

- Eigenvalue equation:

$$H_v \Psi = E\Psi$$

$$H_v \sum_{k=1}^{N_b} x_k \Phi_k = E \sum_{k=1}^{N_b} x_k \Phi_k$$

$$\times \Phi_i^* \qquad (i = 1, 2, \cdots, N_b)$$

$$\sum_{k=1}^{N_b} (H_v)_{ik} x_k = E x_i$$

$$(H_v)_{ik} = \langle \Phi_i | H_v | \Phi_k \rangle$$

$$\langle \Phi_i | \Phi_k \rangle = \delta_{ik}$$

Simultaneous Equation

- N_b-dimentional linear simultaneous equations:

$$\begin{pmatrix} (H_v)_{11} & (H_v)_{12} & \cdots & (H_v)_{1N_b} \\ (H_v)_{21} & (H_v)_{22} & \cdots & (H_v)_{2N_b} \\ \vdots & \vdots & \ddots & \vdots \\ (H_v)_{N_b1} & (H_v)_{N_b2} & \cdots & (H_v)_{N_bN_b} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_b} \end{pmatrix} = E \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_b} \end{pmatrix}$$

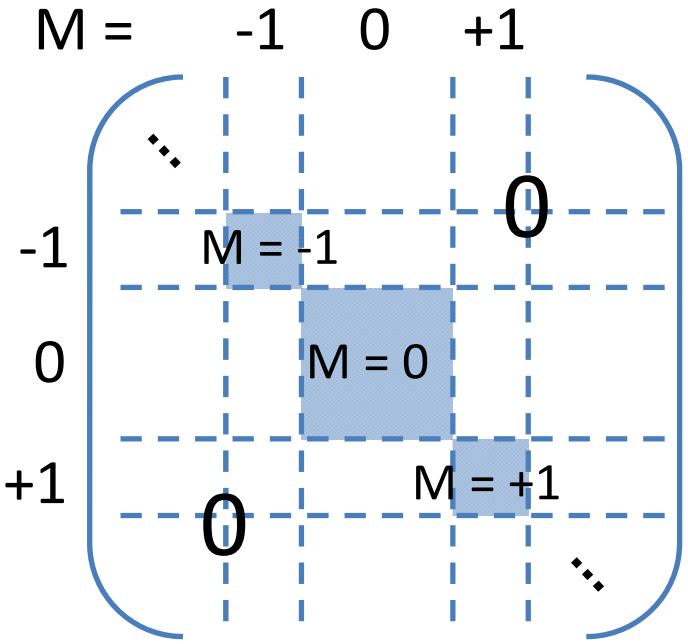
- Diagonalization

$$E = E_1, E_2, \cdots, E_{N_b} \qquad \{x\} = x_1, x_2, \cdots, x_{N_b}$$

Hamiltonian matrix

- Hamiltonian matrix is decomposed into sub matrices belonging to each value of M

$$H =$$



Conservation of M (= Jz) : $M = m_1 + m_2 + \cdots + m_k$

$$[H,\ J_z] = 0$$

What about J?

- An exercise: 2 neutrons in 0f7/2 orbit

J+: Angular momentum raising operator $J_+|J,M\rangle \rightarrow |J,M+1\rangle$

m_1	m_2
7/2	- 7/2
5/2	- 5/2
3/2	- 3/2
1/2	- 1/2
M = 0	

J_+

m_1	m_2
7/2	- 5/2
5/2	- 3/2
3/2	- 1/2
M = 1	

J_+

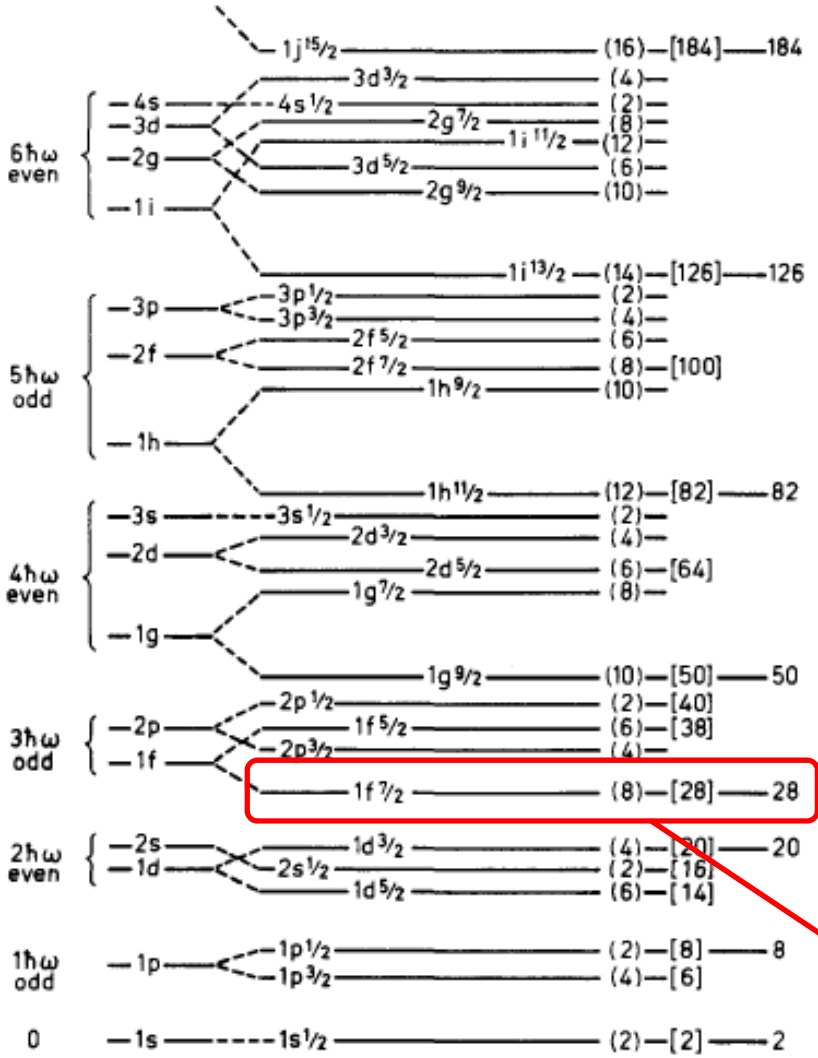
m_1	m_2
7/2	- 3/2
5/2	- 1/2
3/2	1/2
M = 2	

J = 0 two-body state is lost

J = 1 can be eliminated, but is not contained.

Bohr -Mottelson

Single-particle orbits



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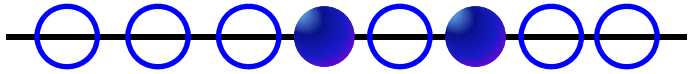
Orbital Angular Momentum	Symbol
l = 0	s
l = 1	p
l = 2	d
l = 3	f
l = 4	g
l = 5	h
l = 6	i

#(sps) = 2J + 1 = 2x7/2 + 1 = 8

m = -7/2, -5/2, ..., 0, ... 7/2

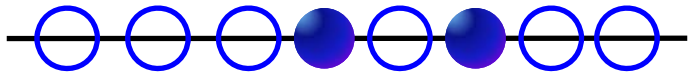
J components in M

2 neutrons in 0f7/2 orbit  0f7/2

	Dimension	J components
M = 0	4	J = 0, 2, 4, 6
M = 1	3	J = 2, 4, 6
M = 2	3	J = 2, 4, 6
M = 3	2	J = 4, 6
M = 4	2	J = 4, 6
M = 5	1	J = 6
M = 6	1	J = 6

M's are conserved w/ the SDs, but J's are mixed for M fixed.

Diagonalization of the Hamiltonian matrix

2 neutrons in 0f7/2 orbit  0f7/2

- By diagonalizing the Hamiltonian matrix, we can obtain wave functions of good J values by superposing Slater determinants (SDs).

$H =$ M = 0

$$\begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \Rightarrow \begin{pmatrix} E(J=0) & 0 & 0 & 0 \\ 0 & E(J=2) & 0 & 0 \\ 0 & 0 & E(J=4) & 0 \\ 0 & 0 & 0 & E(J=6) \end{pmatrix}$$

This property is general,
and valid for the cases with more than 2 particles.

Numerical methods of diagonalization

- Algorithm:
- Householder method for smaller matrix-size
- Lanczos method: for larger matrix-size

Traditional diagonalization method

- Lanczos method:
  - Obtain a few eigenvalues from the lowest energy state
  - Start from the initial vector v_1

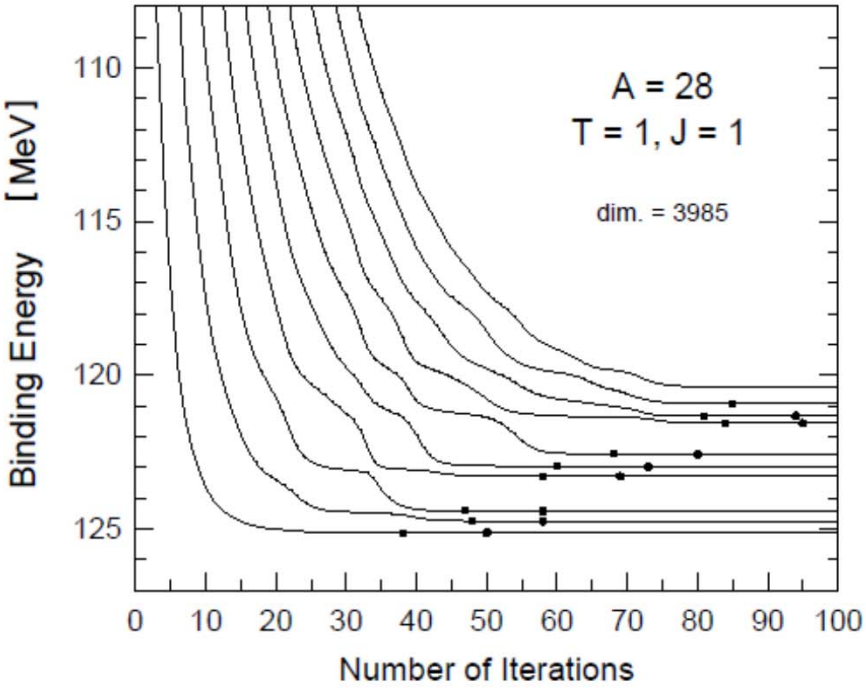
$$Hv_1 = \alpha_1 v_1 + \beta_1 v_2$$
$$Hv_2 = \beta_1 v_1 + \alpha_2 v_2 + \beta_2 v_3$$
$$Hv_3 = \beta_2 v_2 + \alpha_3 v_3 + \beta_3 v_4$$
$$\cdots$$

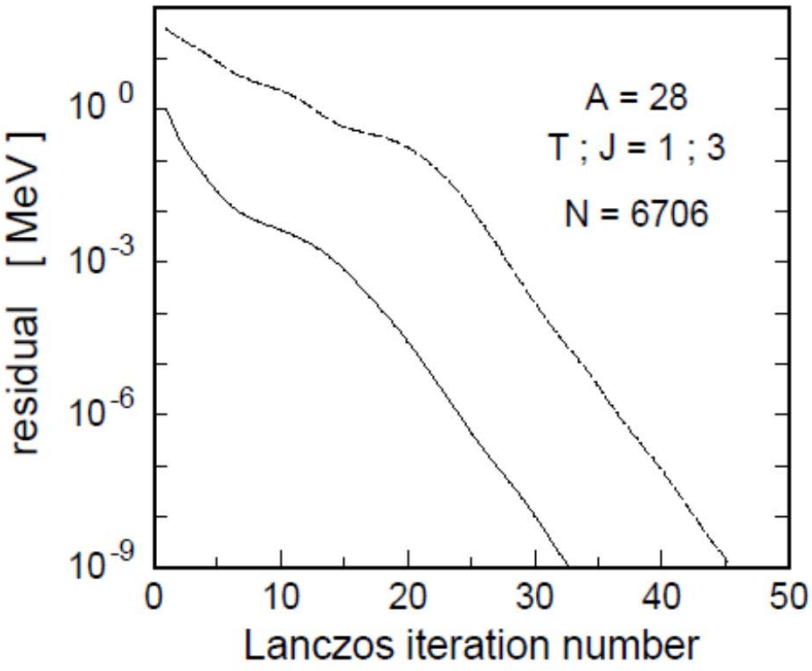
Orthogonalization of Hv_i & v_i , v_{i-1} -> v_{i+1}

  - Diagonalization of the ith tridiagonal matrix

    Convergence of the eigenvalues, even @ i << n

$$J_i = \begin{bmatrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \beta_2 & & \\ & \beta_2 & \ddots & & \\ & & & \alpha_{i-1} & \beta_{i-1} \\ & & & \beta_{i-1} & \alpha_i \end{bmatrix}$$


A = 28
T = 1, J = 1
dim. = 3985
Binding Energy [MeV]
110
115
120
125
0
10
20
30
40
50
60
70
80
90
100
Number of Iterations


A = 28
T ; J = 1 ; 3
N = 6706
residual [MeV]
10 0
10-3
10-6
10-9
0
10
20
30
40
50
Lanczos iteration number

Example (^{18}O = ^{16}O core + 2 n)

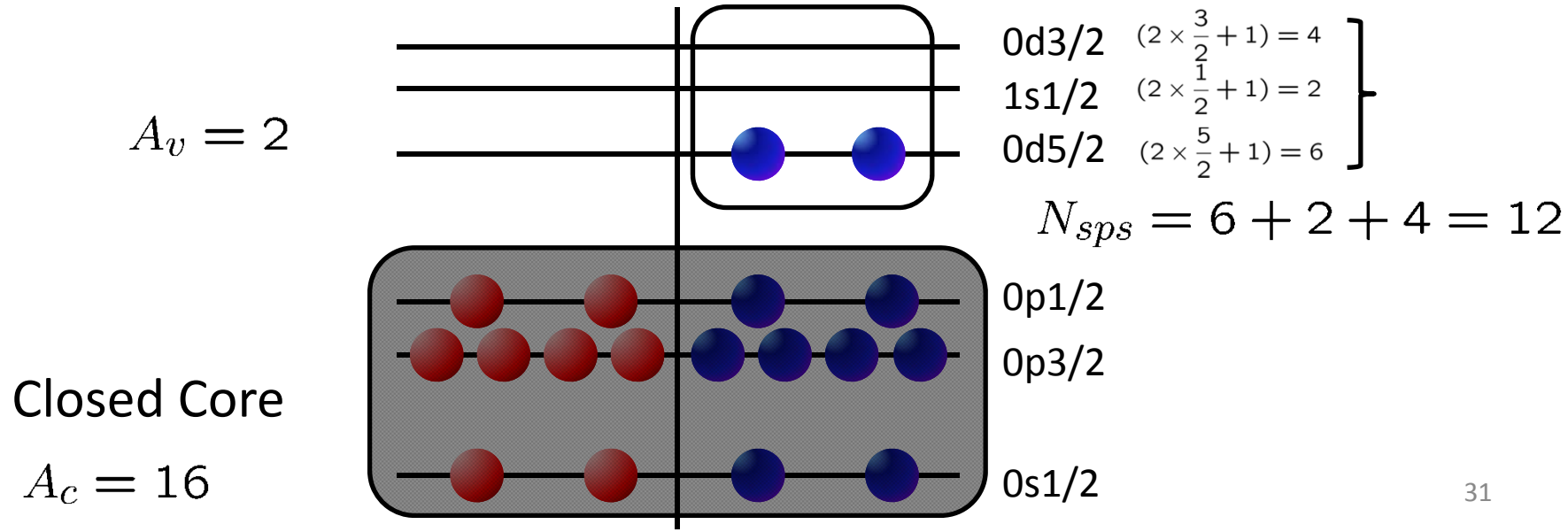
Shell-model caluculation

- Example: 18O (Z = 8, N = 10)
- Assume 16O (Z = 8, N = 8) as the doubly-closed core

  #(Valence Nucleons): $A_v = A - A_c = 18 - 16 = 2$
- Take the sd-shell model space

  #(Single-particle state): $N_{sps} = 12$

Model Space



Single-particle energy

- Single-particle energies: taken from the 17O energy spectra

$$H = H_c + H_v = H_c + (H_0 + V_v)$$

$$H_v = H_0 + V_v$$

$$H_0\psi_i(\mathbf{r}_j) = \epsilon_i\psi_i(\mathbf{r}_j)$$

Experimentally measured values

$$H_c = \sum_{i \le A_c} T_i + \sum_{i<j \le A_c} V_{ij} \qquad H_0 = \sum_{A_c<i} (T_i + U_i) \qquad V_v = \sum_{A_c<i<j} V_{ij}$$

$$U_j = \sum_{i \le A_c} V_{ij}$$

Two-body Matrix Elements

- TBME: Phenomenological Effective Interaction

$$H = H_c + H_v = H_c + (H_0 + V_v)$$

$$H_v = H_0 + V_v$$

Effective Interaction
(determination is explained later)

$$H_c = \sum_{i \le A_c} T_i + \sum_{i<j \le A_c} V_{ij} \qquad H_0 = \sum_{A_c<i} (T_i + U_i) \qquad V_v = \sum_{A_c<i<j} V_{ij}$$

$$U_j = \sum_{i \le A_c} V_{ij}$$

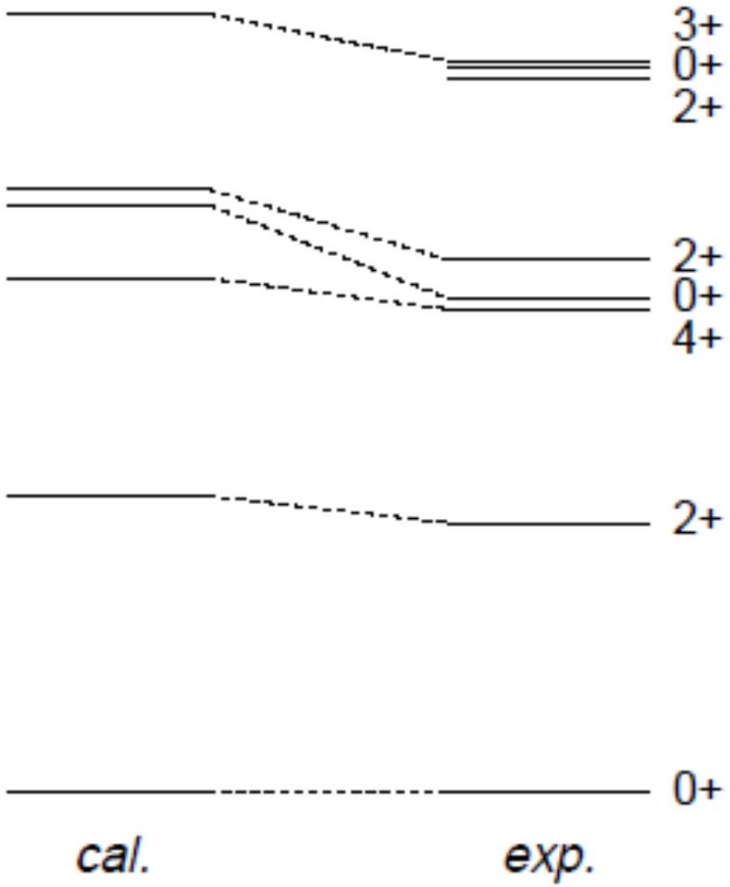
Basis state

- #(Slater determinant): $\begin{pmatrix} N_{sps} \\ A_v \end{pmatrix} = \begin{matrix} 12 \\ 2 \end{matrix} = 66$
- Conservation of M: $M = m_1 + m_2$
- For M = 0, $N_b = 14$
- Possible M = 0 2-particle states $J \geq M$

	(j1, m1)	(j2, m2)		(j1, m1)	(j2, m2)
1	(5/2, +5/2)	(5/2, -5/2)	8	(5/2, -1/2)	(3/2, +1/2)
2	(5/2, +3/2)	(5/2, -3/2)	9	(5/2, -3/2)	(3/2, +3/2)
3	(5/2, +1/2)	(5/2, -1/2)	10	(1/2, +1/2)	(1/2, -1/2)
4	(5/2, +1/2)	(1/2, -1/2)	11	(1/2, +1/2)	(3/2, -1/2)
5	(5/2, -1/2)	(1/2, +1/2)	12	(1/2, -1/2)	(3/2, +1/2)
6	(5/2, +3/2)	(3/2, -3/2)	13	(3/2, +3/2)	(3/2, -3/2)
7	(5/2, +1/2)	(3/2, -1/2)	14	(3/2, +1/2)	(3/2, -1/2)

18O energy spectra

- Diagonalization of 14x14 Hamiltonian matrix



Some remarks

1. TBMEs
2. Determination of TBMEs
3. Scheme

TBMEs

Two-body matrix elements (TBMEs)

- A two-body state is written as

$$|j_1, j_2, J, M\rangle = \sum_{m_1, m_2} \langle j_1, m_1, j_2, m_2 | J, M\rangle |j_1, m_1\rangle |j_2, m_2\rangle$$

- Two-body matrix elements (TBMEs)

$$\begin{aligned}\langle j_1, j_2, J, M | V | j_3, j_4, J', M'\rangle \;=\; & \sum_{m_1, m_2} \langle j_1, m_1, j_2, m_2 | J, M\rangle \\ & \times \sum_{m_1, m_2} \langle j_1, m_1, j_2, m_2 | J', M'\rangle \\ & \times \langle j_1, m_1, j_2, m_2 | V | j_3, m_3, j_4, m_4\rangle |j_2, m_2\rangle\end{aligned}$$

TBMEs can be non-zero, only if J = J' & M = M',
because V is rotationally invariant (a scalar w.r.t. the rotation).

$$\left[V, J^2\right] = [V, J_z] = 0$$

Two-body matrix elements (TBMEs)

- Two-body matrix elements (TBMEs)

$$\langle j_1, j_2, J, M | V | j_3, j_4, J, M \rangle$$

are independent of M values, also because V is rotationally invariant.

$$\langle j_1, j_2, J | V | j_3, j_4, J \rangle \qquad \left[V, J^2\right] = [V, J_z] = 0$$

- Two-body matrix elements are assigned by

j_1, j_2, j_3, j_4 and J

Because of the complexity of nuclear force, one cannot express all TBMEs by few empirical parameters.

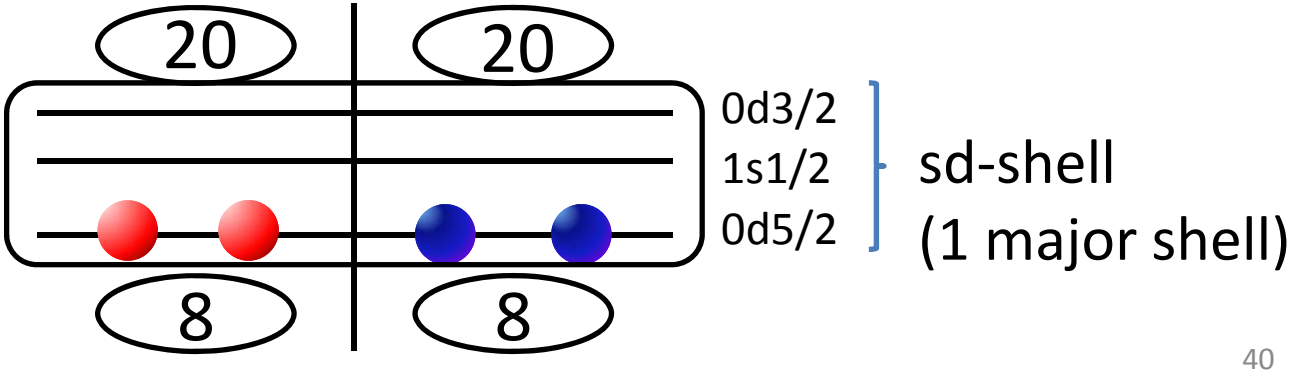
Example of the TBMEs

- An example of TBMEs: USD interaction (Wildenthal & Brown)

- Model space: sd-shell (d5/2, d3/2 and s1/2)
- 3 single-particle energies (SPEs): $\epsilon_{0d5/2}$, $\epsilon_{0d3/2}$ and $\epsilon_{1s1/2}$
- 63 TBMEs: $\langle j_1, j_2, J, T|V|j_3, j_4, J, T\rangle$

  (Isospin is also a good quantum #)

Model Space
(Valence Space)



USD interaction

i	j	k	l	J	T	v
1	1	1	1	0	1	-2.1845
1	1	1	1	1	0	-1.4151
1	1	1	1	2	1	-0.0665
1	1	1	1	3	0	-2.8842
2	1	1	1	1	0	0.5647
2	1	1	1	2	1	-0.6149
2	1	1	1	3	0	2.0337
2	1	2	1	1	0	-6.5058
2	1	2	1	1	1	1.0334
2	1	2	1	2	0	-3.8253
2	1	2	1	2	1	-0.3248
2	1	2	1	3	0	-0.5377
2	1	2	1	3	1	0.5894
2	1	2	1	4	0	-4.5062
2	1	2	1	4	1	-1.4497
2	1	3	1	1	0	-1.7080
2	1	3	1	1	1	0.1874
2	1	3	1	2	0	0.2832
2	1	3	1	2	1	-0.5247
2	1	3	3	1	0	2.1042
2	2	1	1	0	1	-3.1856
2	2	1	1	1	0	0.7221
2	2	1	1	2	1	-1.6221
2	2	1	1	3	0	1.8949
2	2	2	1	1	0	2.5435
2	2	2	1	2	1	-0.2828
2	2	2	1	3	0	2.2216
2	2	2	1	4	1	-1.2363
2	2	2	2	0	1	-2.8197
2	2	2	2	1	0	-1.6321
2	2	2	2	2	1	-1.0020
2	2	2	2	3	0	-1.5012

⋮

i,j,k,l

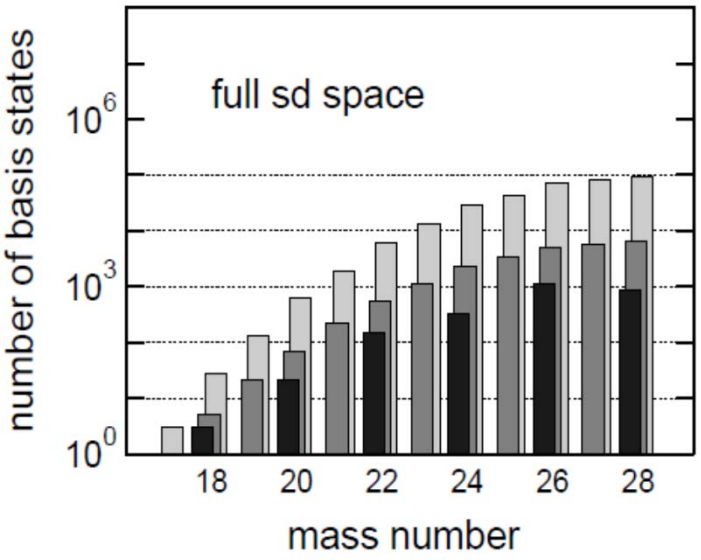
1: 0d3/2

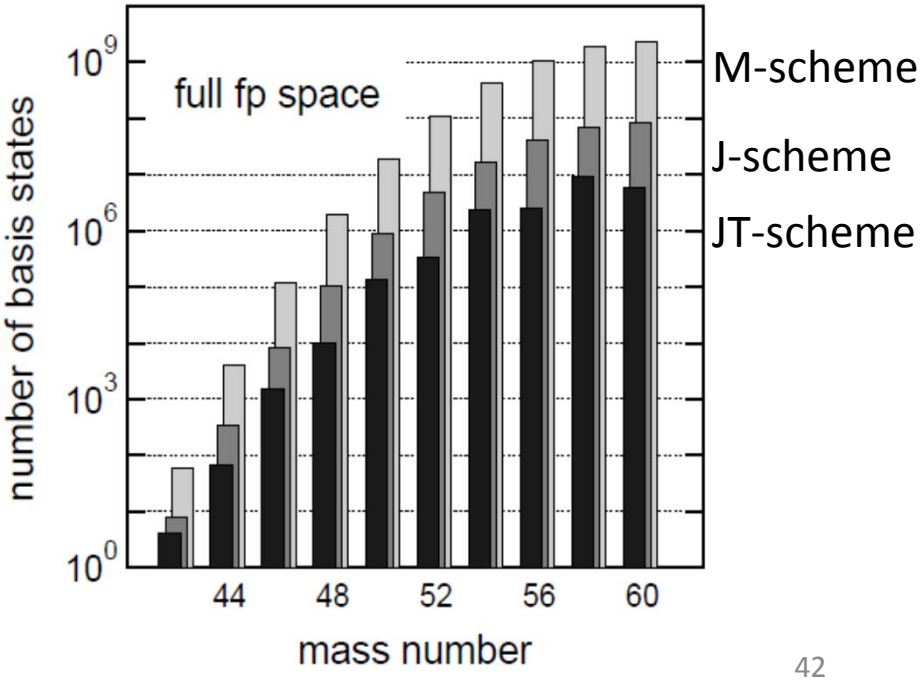
2: 0d5/2

3: 1s1/2

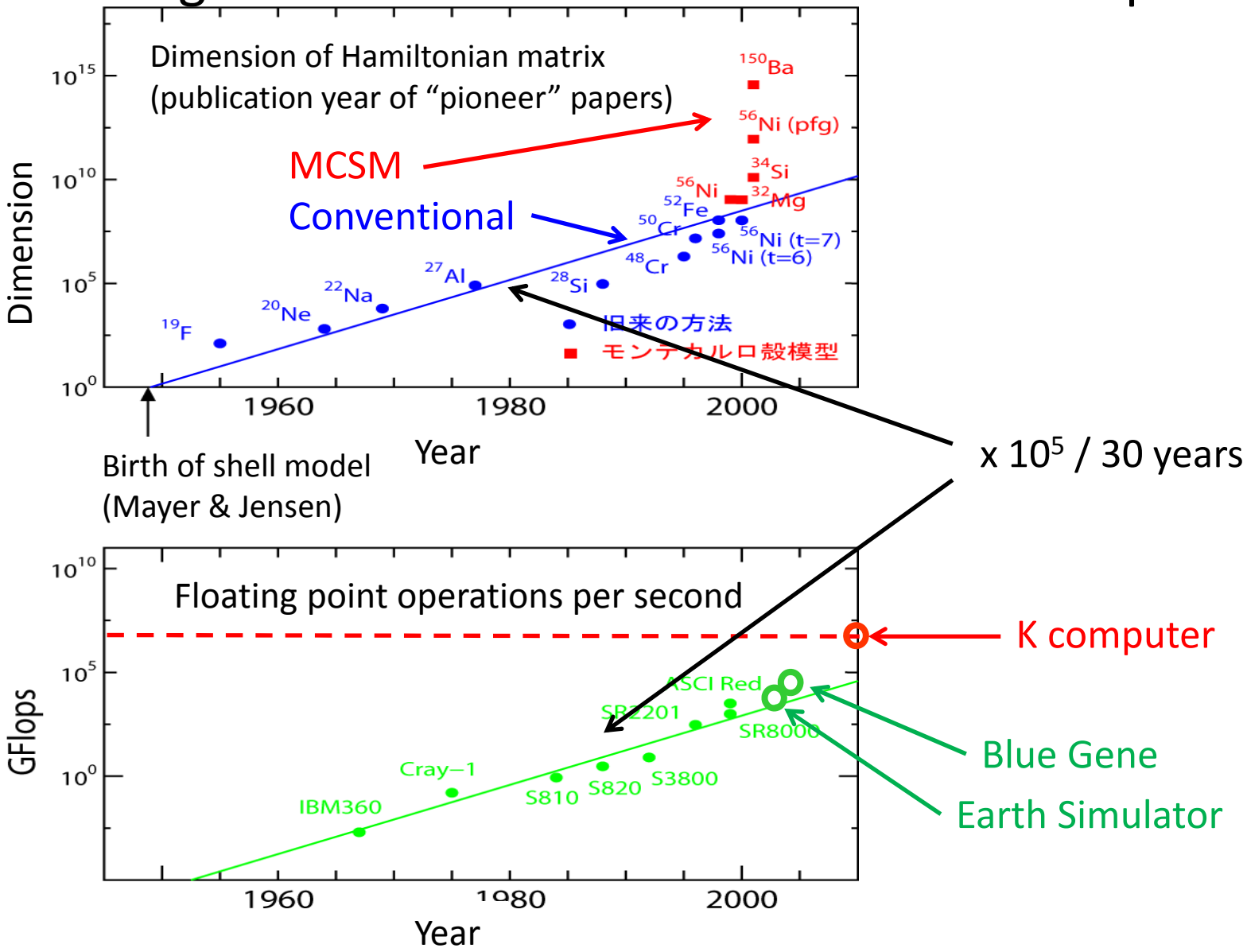
Dimensions

- Dimension: Maximum # of the basis
- sd-shell: $8 < (Z, N) < 20$ ($(sd)^n$ configuration)
- pf-shell: $20 < (Z, N) < 40$ ($(pf)^n$ configuration)



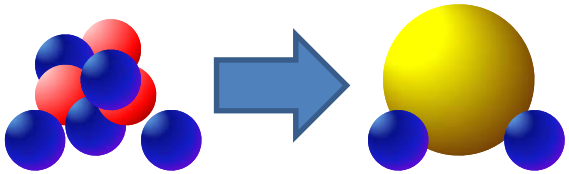


Progress in shell-model calculations & computers



Determination of the TBMEs

Shell-Model Hamiltonian



$$H = \sum_{i=1}^{N_{sps}} \epsilon_i n_i + \sum_{ijkl}^{N_{sps}} v_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$$

$$n_i = a_i^\dagger a_i$$

ϵ_i : Single-particle energy

v_{ijkl} : Two-body matrix element (TBME)

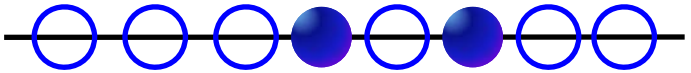
$$\langle j_1, j_2, J, T | V | j_3, j_4, J, T \rangle$$

Determination of the TBMEs

- Early time: Closed shell + Experimental levels of 2 valence particles

TBME

- Example: 0^+, 2^+, 4^+, 6^+ in ^{42}Ca: well isolated 0f7/2 orbit



$V_J = \langle 0f7/2, 0f7/2, J, T=1 | V | 0f7/2, 0f7/2, J, T=1 \rangle$

are determined directly by the experimental observations.

Experimental energy of state J

$E(J) = 2\,\varepsilon(0f7/2) + V_J$

Experimental single-particle energy of 0f7/2

Bohr -Mottelson

Single-particle orbits

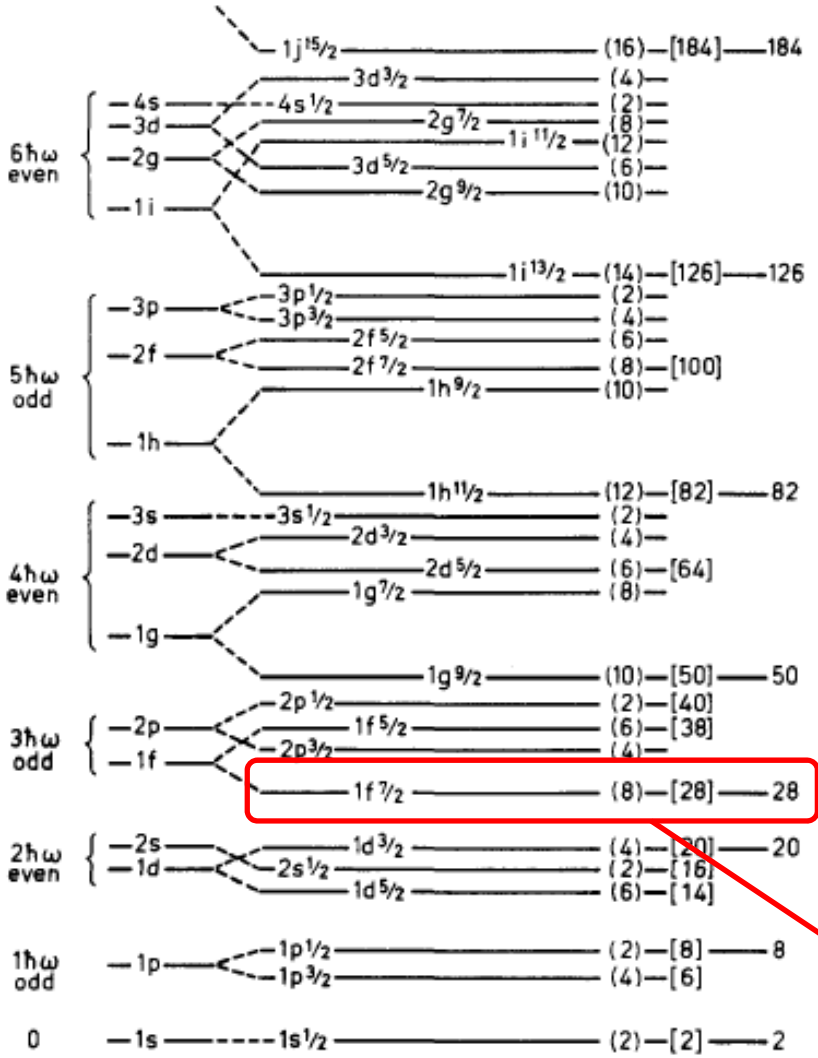


Figure 2-23 Sequence of one-particle orbits. The figure is taken from M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

NL_J

$N = 2n + l\ (+ 1)$

$= 1, 2, 3, \ldots$ $\qquad J = l + s$

$(= 0, 1, 2, \ldots)$

Orbital Angular Momentum	Symbol
l = 0	s
l = 1	p
l = 2	d
l = 3	f
l = 4	g
l = 5	h
l = 6	i

#(sps) = 2J + 1 = 2x7/2 + 1 = 8

m = -7/2, -5/2, ..., 0, ... 7/2

Empirical determination

- The isolation of 0f7/2 is special. In the other cases, several orbits must be taken into account.

- In general, chi^2 fit is made as follows,

  (1) TBMEs are assumed.

  (2) chi^2 fit is calculated btw theor. & exp. energy levels.

  (3) TBMEs are modified.

  (4) Go to (1), & iterate (1)-(3) until chi^2 becomes small enough

- Example: 0^+, 2^+, 4^+ in ^{18}O: 0d5/2 & 1s1/2

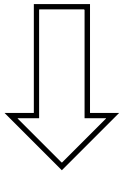
$$\langle 0d5/2, 0d5/2, J, T = 1|V|0d5/2, 0d5/2, J, T = 1\rangle$$
$$\langle 0d5/2, 1s1/2, J, T = 1|V|0d5/2, 0s1/2, J, T = 1\rangle$$
$$\vdots$$

Arima, Cohen, Lawson & McFarlane (1968)

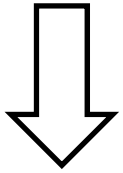
- At the beginning, it was a perfect chi^2 fit.
- As heavier nuclei are studied,

  (1) the number of TBMEs increases,

  (2) shell-model calculations become huge



- Complete fit becomes more difficult and finally impossible



Hybrid version

Hybrid version

- Microscopically calculated TBMEs
  eg.) G-matrix (Kuo-Brown, H.-Jensen, …)

  G-matrix based TBMEs are not perfecxt
  direct use for shell model calculation is only disaster

- Use G-matrix-based TBMEs as a starting point of the chi^2 fit, and do the chi^2 fit to the experiments.
  (Consider some linear combinations of TBMEs and fit them)

cont'd

- The chi^2 fit method produces, as a result of minimization,
  a set of linear equations of TBMEs

- Some linear combinations of TBMEs are sensitive
  to available experimental data (ground & low-lying states).

  The others are insensitive. Those are assumed to be given by G-matrix-based calculation (i.e. no fit).

- Firstly done for sd-shell: Wildenthal & Brown's USD interaction
  47 linear combinations (1970)

  Recent version of USD: G-matrix-based TBMEs have been improved
  -> 30 linear combinations fitted

Example

- An example from pf-shell (f7/2, f5/2, p3/2, p1/2)

  G-matrix + polarization correction + empirical refinement

  Microscopic Empirical (Phenomenological)

- GXPF1 interaction: M. Honma et al., PRC65 (2002) 061301(R)
- start from a realistic microscopic interaction

➢ M. Hjorth-Jensen, et al., Phys. Rept. 261 (1995) 125

- Bonn-C potential
- 3rd order Q-box + folded diagram

✓ 195 TBMEs & 4 SPEs are calculated -> not completely good (theory imperfect)

➢ Vary 70 linear combinations of 195 TBME2 & 4 SPEs

➢ Fit to 699 exp. energy data of 87 nuclei

Bohr -Mottelson

Single-particle orbits

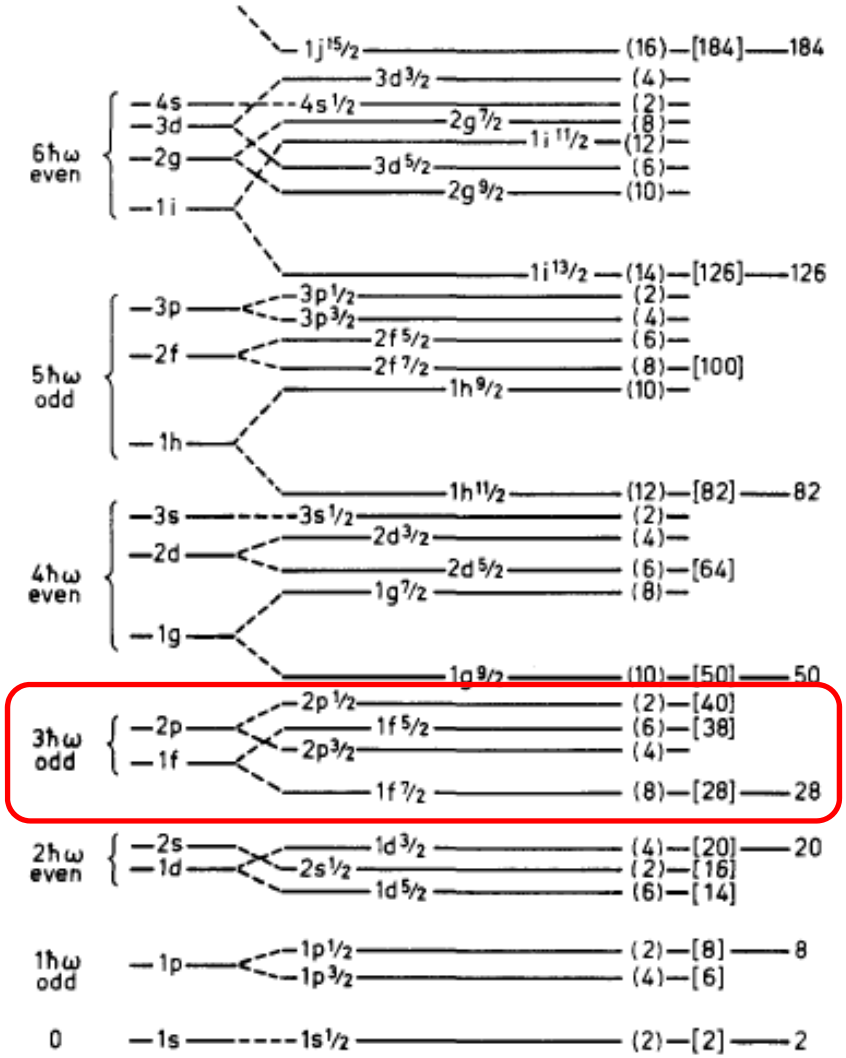


Figure 2-23 Sequence of one-particle orbits. The figure is taken from M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955.

NL_J

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$= 1, 2, 3, \ldots$ $\quad J = l + s$

$(= 0, 1, 2, \ldots)$

Orbital Angular Momentum	Symbol
l = 0	s
l = 1	p
l = 2	d
l = 3	f
l = 4	g
l = 5	h
l = 6	i

pf-shell

#(sps) = 20

G-matrix vs. GXPF1A

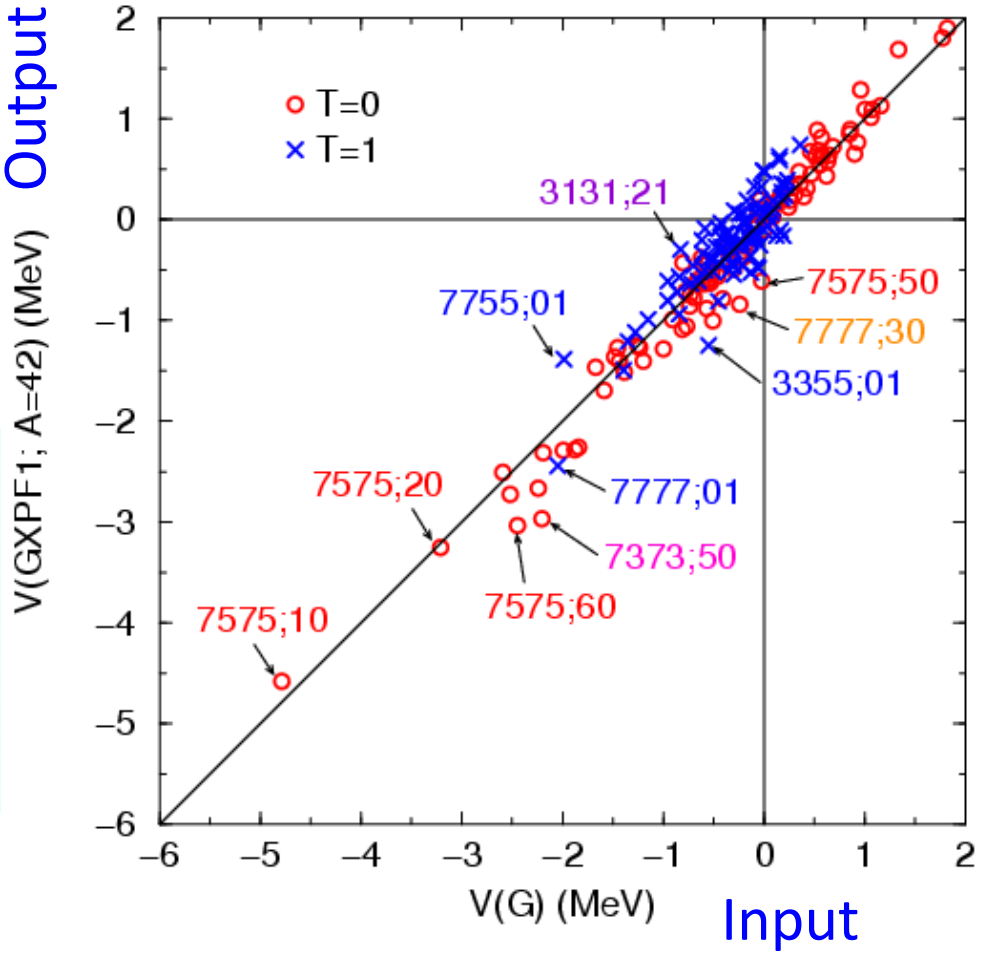
- TBMEs: $\langle ab; JT \mid V \mid cd; JT \rangle$

$7 = f_{7/2}$, $3 = p_{3/2}$, $5 = f_{5/2}$, $1 = p_{1/2}$

- T=0 … attractive
- T=1 … repulsive
- Relatively large modifications in

  $V(abab; J0)$ with large J

  $V(aabb; J1)$ pairing



Scheme

Prescriptions of Shell-model calc.

- M-scheme
  - basis function: Slater determinant
  - SD does not hold J as a good quantum #, but eigen function after the diagonalization holds J as a good quantum #.

$$\left[H_v, J^2\right] = [H_v, J_z] = 0$$

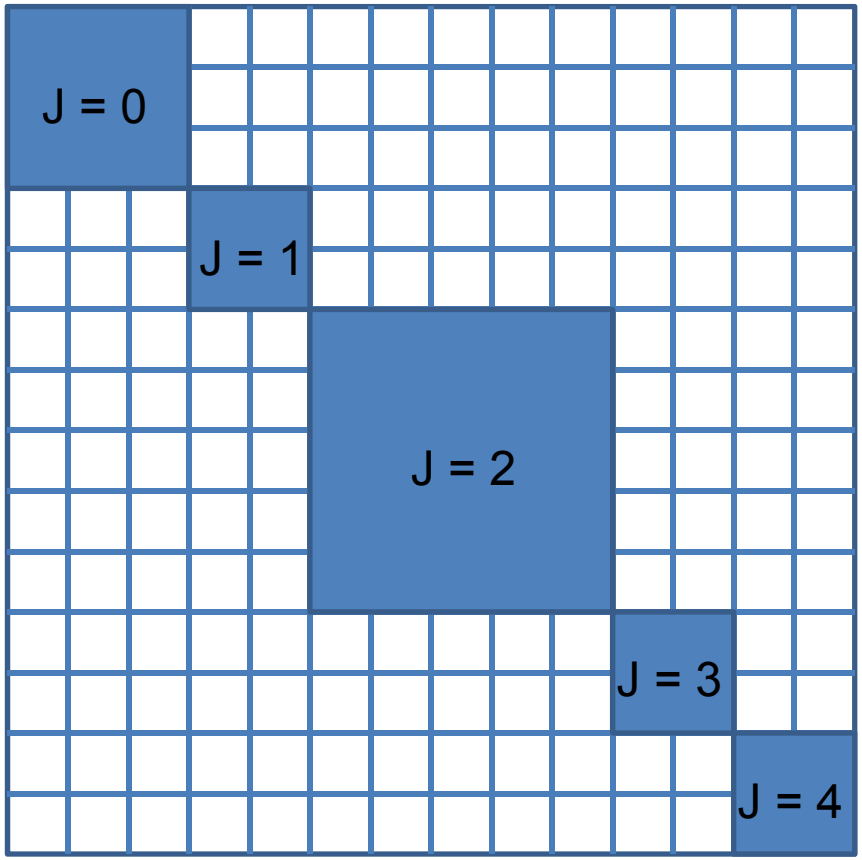
- J-scheme
  - basis function: $\Phi_{JM} = \sum_{m_1,m_2} \langle j_1 m_1 j_2 m_2 | JM \rangle \psi_{j_1 m_1} \psi_{j_2 m_2}$

$$\left[H, J^2\right] = 0$$

- JT-scheme $\left[H, T^2\right] = 0$
- M-scheme + J-projection
- …

J-scheme

- M = 0



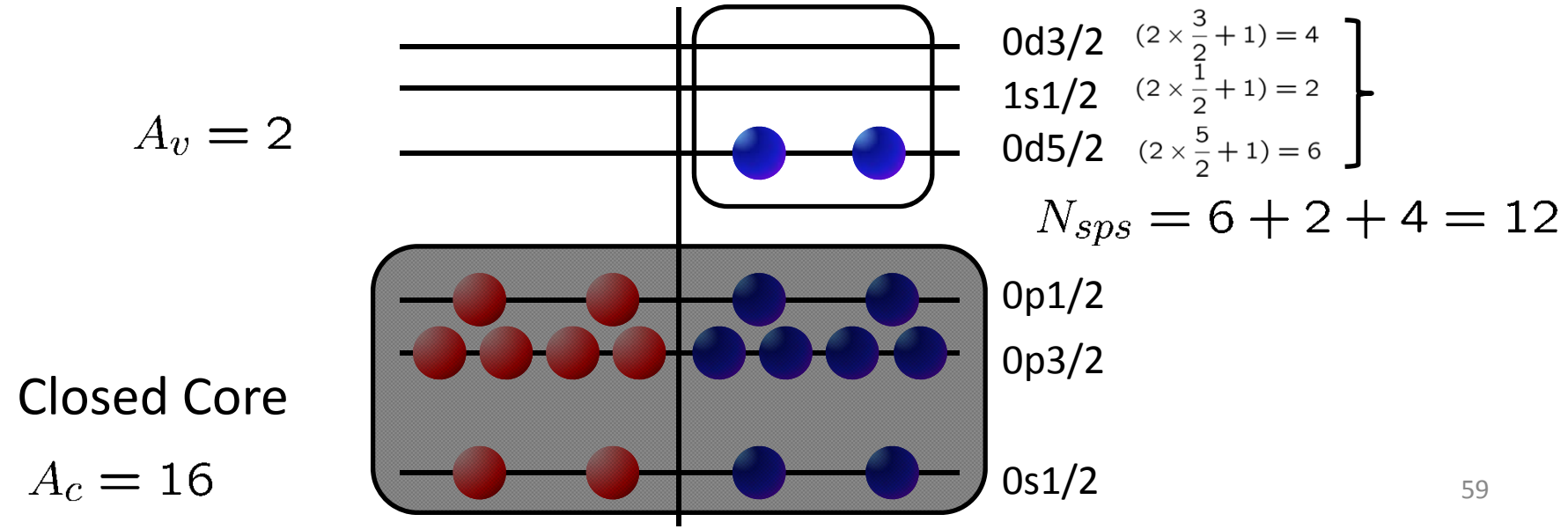
Configuration	J = 0	J = 1	J = 2	J = 3	J = 4
$(d5/2)^2$	✓		✓		✓
(d5/2)(s1/2)			✓	✓	
$(s1/2)^2$	✓				
(d5/2)(d3/2)		✓	✓	✓	✓
(s1/2)(d3/2)		✓	✓		
$(d3/2)^2$	✓		✓		

Shell-model caluculation

- Example: 18O (Z = 8, N = 10)
- Assume 16O (Z = 8, N = 8) as the doubly-closed core

  #(Valence Nucleons): $A_v = A - A_c = 18 - 16 = 2$
- Take the sd-shell model space

  #(Single-particle state): $N_{sps} = 12$



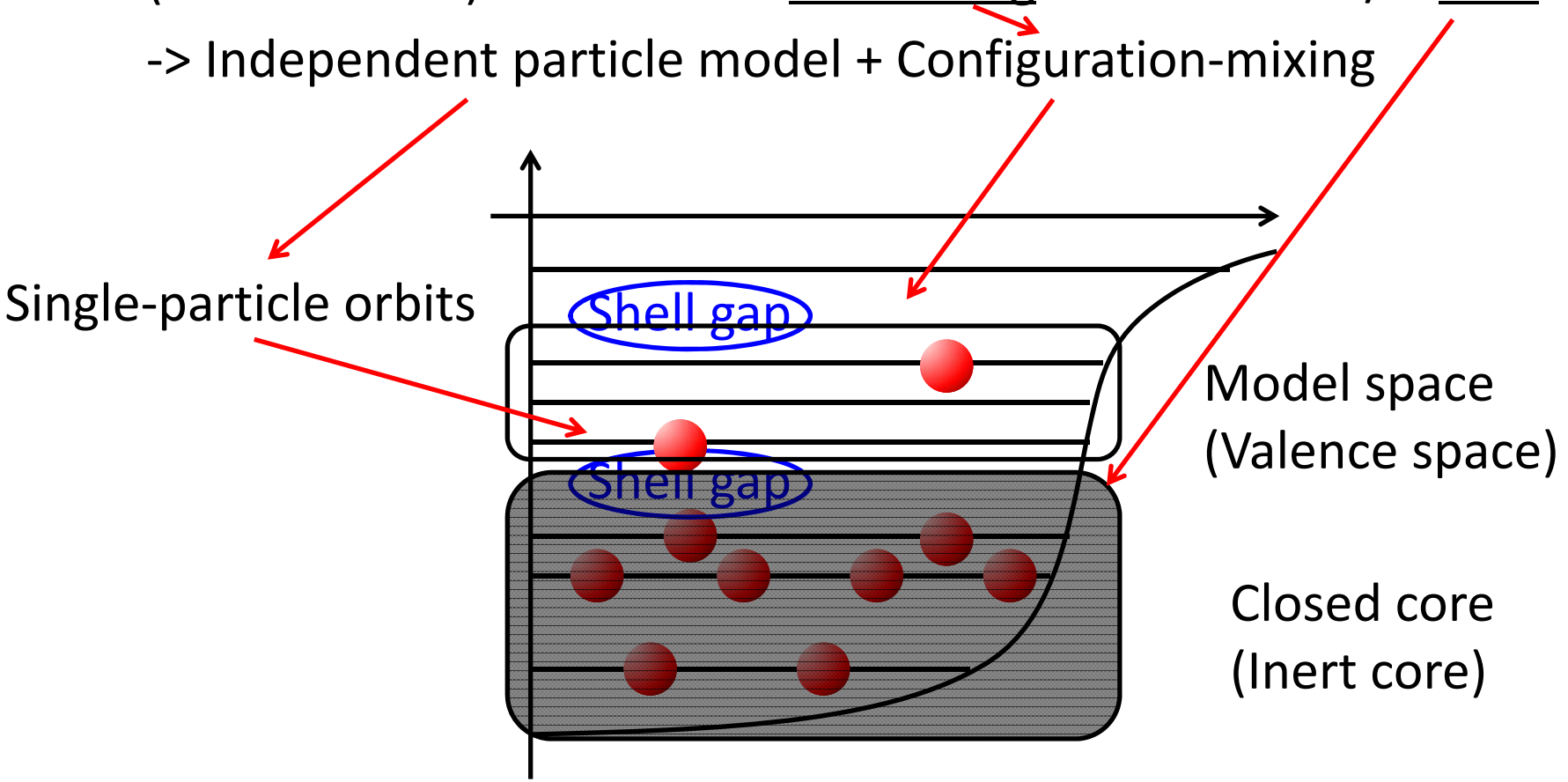
Basis state

- #(Slater determinant): $\begin{pmatrix} N_{sps} \\ A_v \end{pmatrix} = \begin{matrix} 12 \\ 2 \end{matrix} = 66$
- Conservation of M: $M = m_1 + m_2$
- For M = 0, $N_b = 14$
- Possible M = 0 2-particle states $J \geq M$

	(j1, m1)	(j2, m2)		(j1, m1)	(j2, m2)
1	(5/2, +5/2)	(5/2, -5/2)	8	(5/2, -1/2)	(3/2, +1/2)
2	(5/2, +3/2)	(5/2, -3/2)	9	(5/2, -3/2)	(3/2, +3/2)
3	(5/2, +1/2)	(5/2, -1/2)	10	(1/2, +1/2)	(1/2, -1/2)
4	(5/2, +1/2)	(1/2, -1/2)	11	(1/2, +1/2)	(3/2, -1/2)
5	(5/2, -1/2)	(1/2, +1/2)	12	(1/2, -1/2)	(3/2, +1/2)
6	(5/2, +3/2)	(3/2, -3/2)	13	(3/2, +3/2)	(3/2, -3/2)
7	(5/2, +1/2)	(3/2, -1/2)	14	(3/2, +1/2)	(3/2, -1/2)

Summary

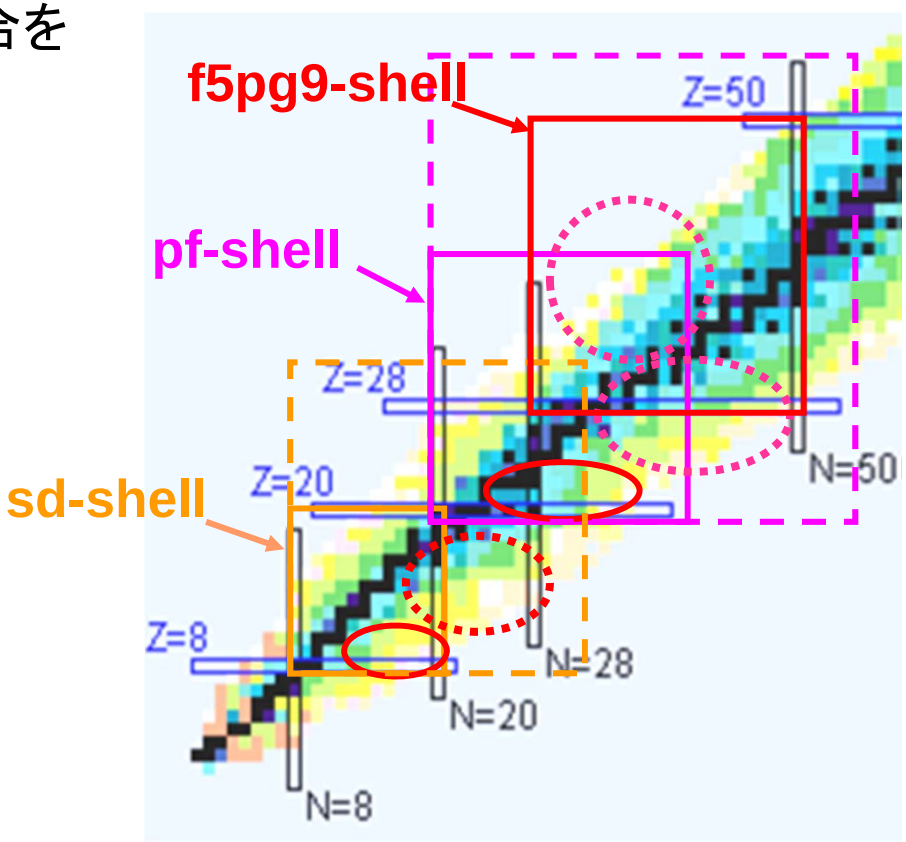
- (Conventional) Shell model: Interacting shell model w/ a core
  -> Independent particle model + Configuration-mixing



Slide from M. Honma

さらに遠くへ

- f5pg9-殻　JUN45相互作用
  - 137パラメータ中45線形結合を400データにフィット
  - 平均誤差185keV
  - g9/2軌道（異パリティ）の影響
- sd-pf　殻
  - ^{40}Ca閉殻の構造
  - 重いsd殻中性子過剰核
- pf-sdg　殻
  - N＝Z近傍核の陽子過剰核
  - 変形共存と形状相転移
  - 重いpf殻中性子過剰核
  - ^{100}Snの閉殻構造



End