有限温度格子QCD 入門

ゼロからの格子ＱＣＤ入門

-- 有限バリオン密度系の研究を目指して --

素核宇宙融合 レクチャーシリーズ」第９回

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もともとは経路積分を計算していた

$$Z = \langle F | e^{iHt} | I \rangle$$

（N等分して、間に完全系を入れて・・・）

$$= \int DAD\bar{\psi} D\psi \, e^{i\iiint dxdydzdtL}$$

（ユークリッド化して）

$$= \int DAD\bar{\psi} D\psi \, e^{-S}$$

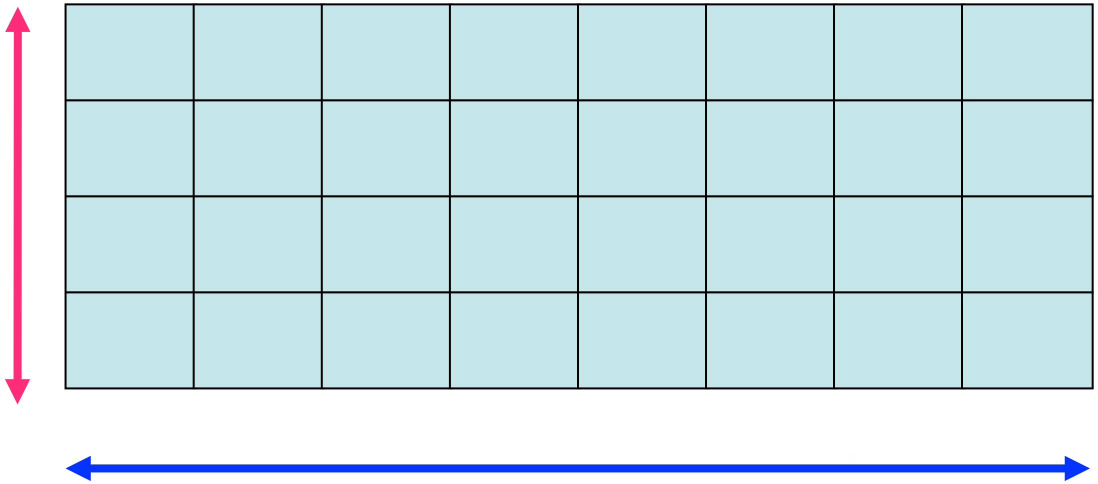
ところがこれは分配関数を経路積分で書いたものと同じ！

$$Z = \mathrm{Tr}\, e^{-\beta H}$$ 但し $$\beta = \frac{1}{kT}$$

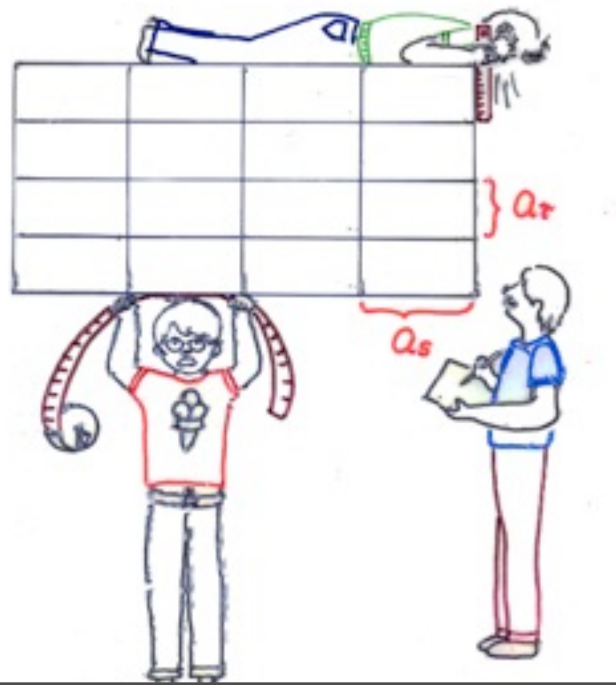
$$\int_{-\infty}^{+\infty} d\tau \Rightarrow \int_0^{\beta} d\tau$$

Finite Temperature

$$N_t a_t = \frac{1}{kT}$$



$$N_s a_s$$



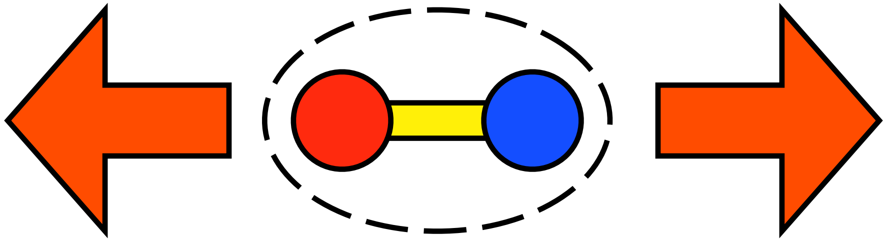
$a_t < a_s$ の時、 anisotropic lattice

Burgers, Karsch, Nakamura and Stamatescu
QCD on anisotropic lattices
Nucl.Phys. B204, pp587--600, 1988

⇨ fine resolution along temperature direction

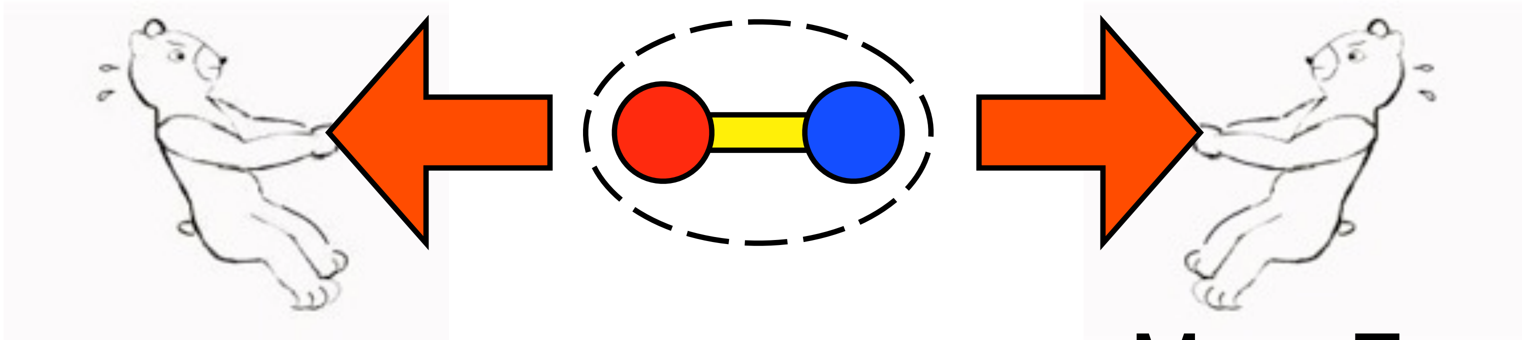
なぜ有限温度QCD？

Confinement



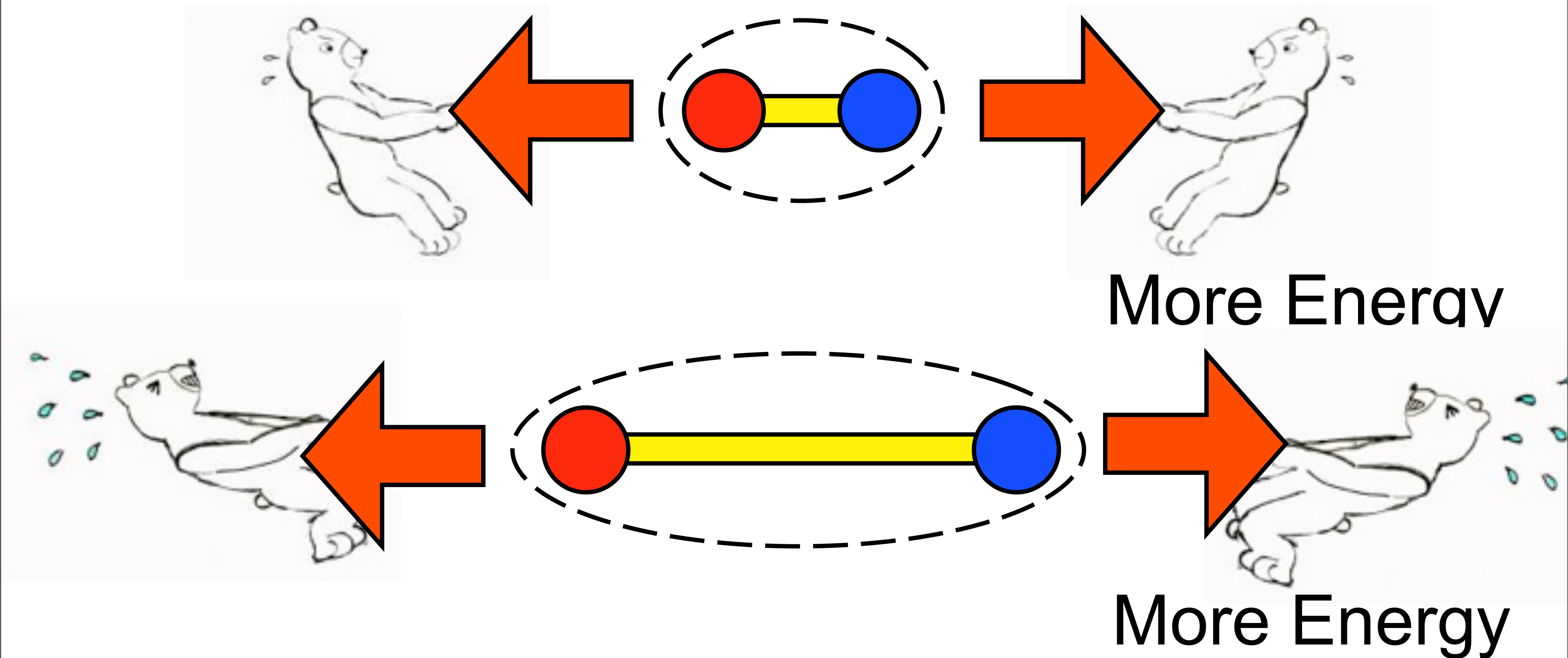
More Energy

Confinement

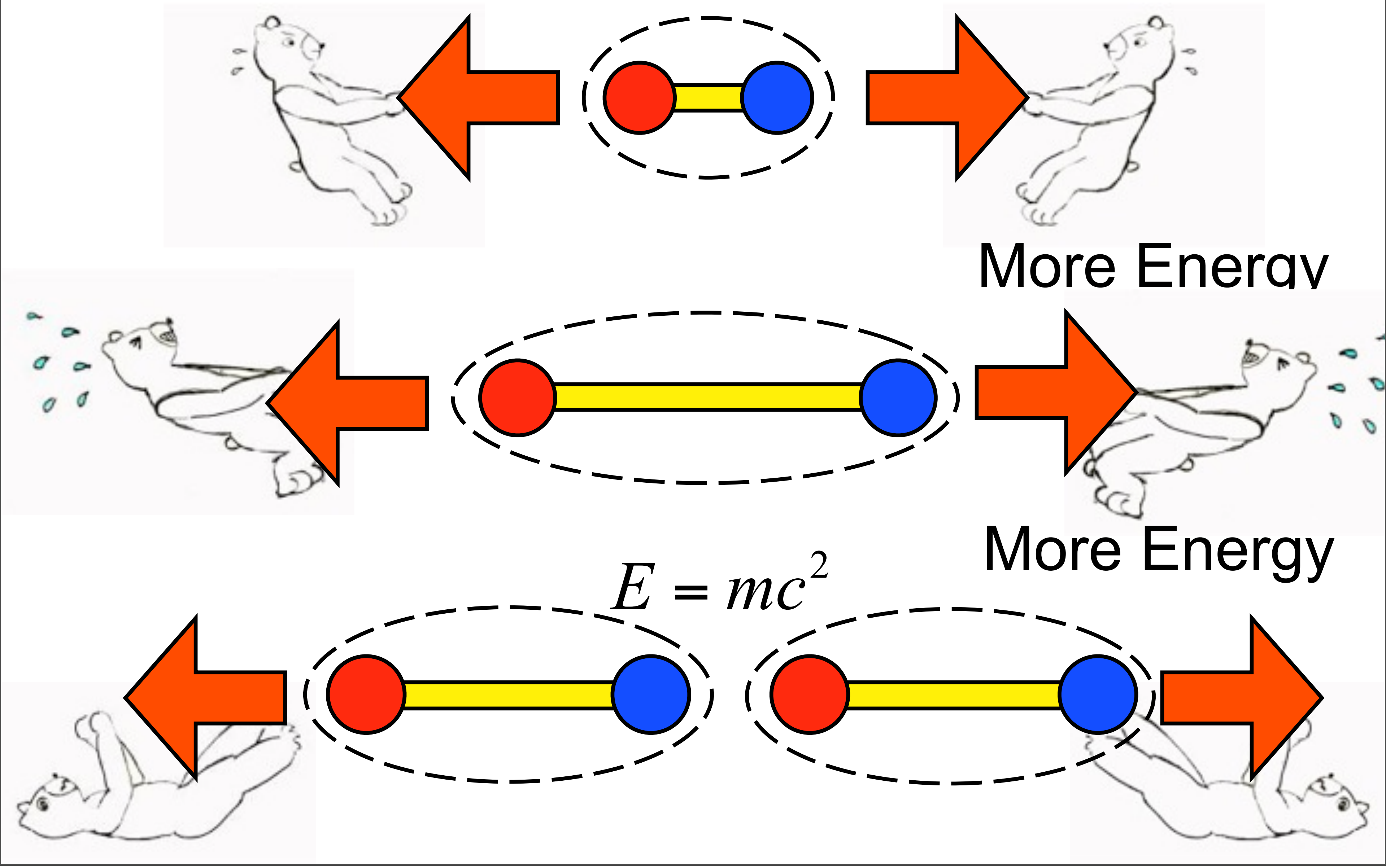


More Energy

Confinement

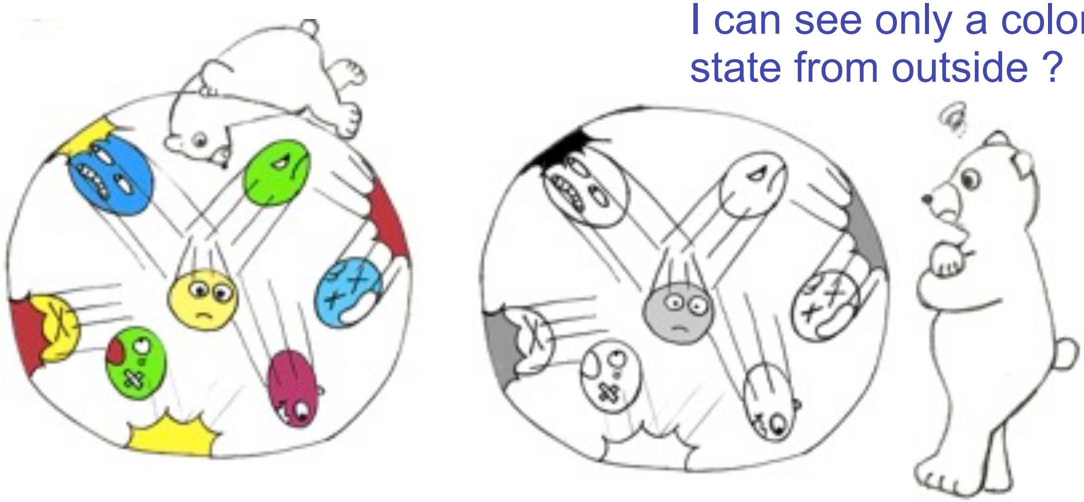


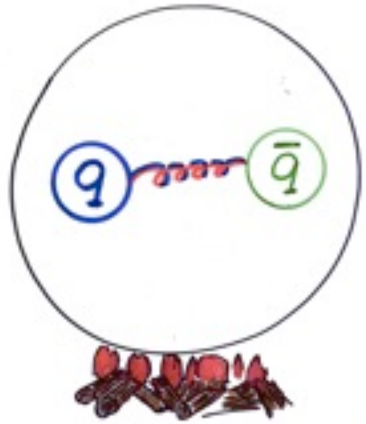
Confinement

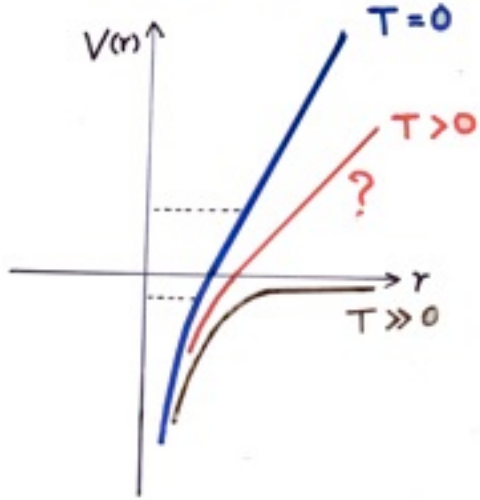


Confinement (2)

I can see only a colorless state from outside ?







Confinement Potential is "screened" at finite temperature.

Deconfinement

Observation of a Phase Transition at Finite Temperature on the Lattice

1981, McLerran and Svetitsky, Kuti, Polonyi and Szlachanyi, Engels et al.

$$Z = e^{-\beta F} = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \sum_{\phi} \left\langle \phi \left| e^{-\beta(H-\mu N)} \right| \phi \right\rangle$$

$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+A Static Quark})}{Z(\text{Gluons})} = \left\langle L(\vec{x}) \right\rangle$$

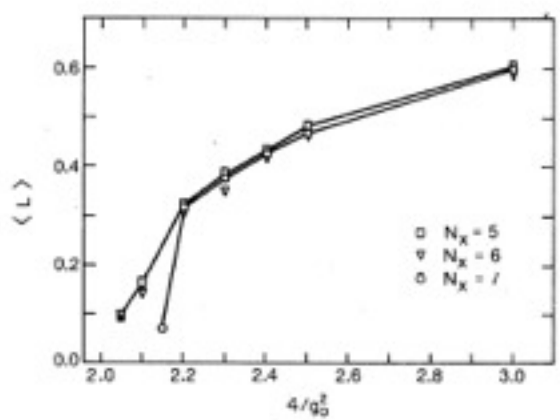
Excess Energy when a quark exists.

$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+Static Quark+Anti-Quark})}{Z(\text{Gluons})}$$

$$= \left\langle L(\vec{x}) L^{\dagger}(\vec{y}) \right\rangle$$

Excess Energy when a quark and an anti-quark exist.

⇨ Heavy Quark Potential



McLerran and Svetitsky, PRD24,（１９８１）

格子上の熱力量の計算

$$Z = e^{-\beta F} = e^{-F/T}$$

$$F = -T \log Z$$

$$\frac{\partial F}{\partial X}$$

X: 体積,化学ポテンシャル , ...

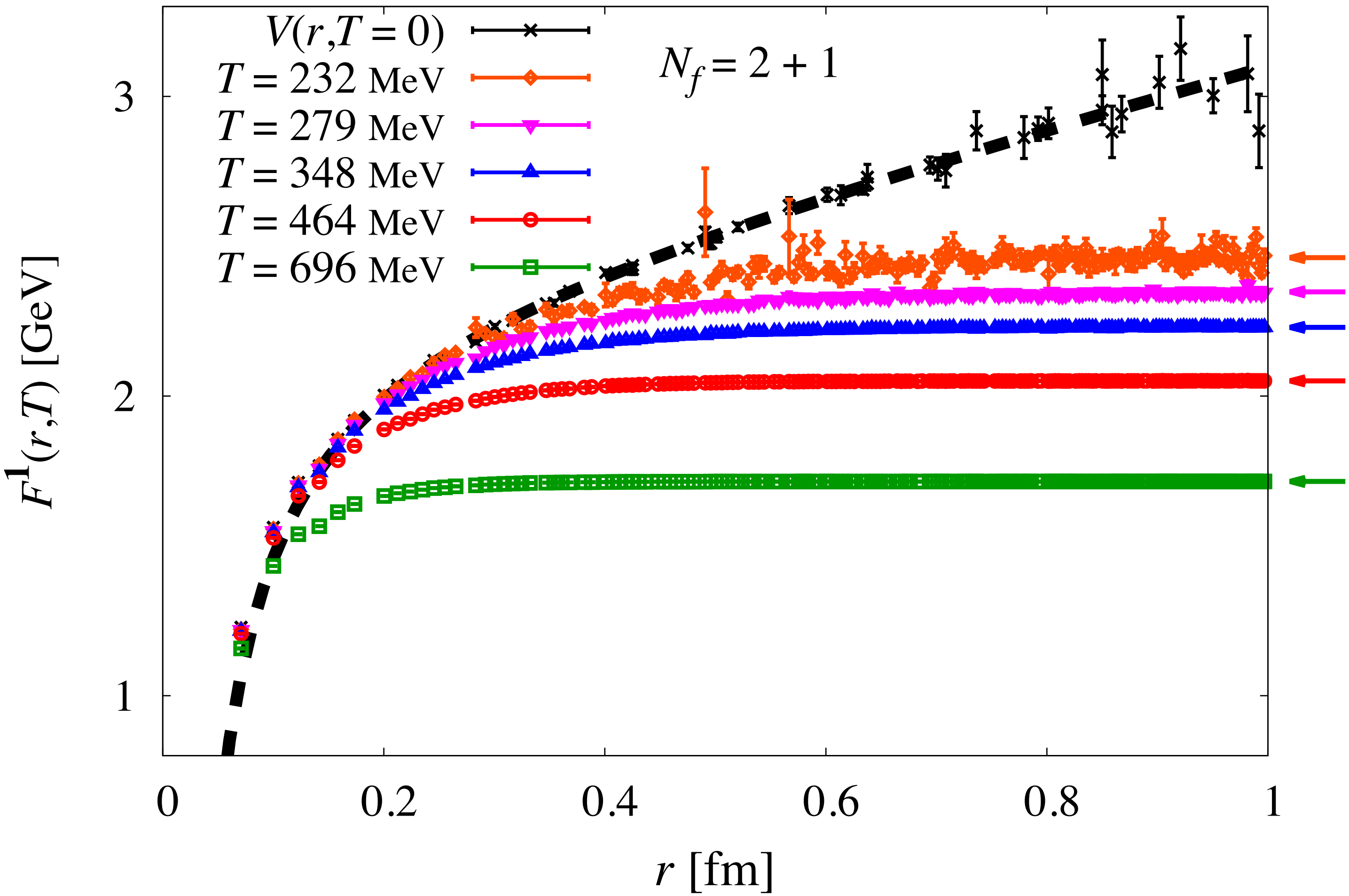
$$Z = \int \mathcal{D}U e^{-S}$$

とすると

$$\frac{\partial}{X} \log Z = -\frac{\int \mathcal{D}U \frac{\partial S}{\partial X} e^{-S}}{Z} = \langle \frac{\partial S}{\partial X} \rangle$$

XがTの時は(格子間隔とTは独立ではないため）注意が必要

Heavy Quark Potential with Dynamical Quarks



Maezawa et al (WHOT-Collaboration) Prog. Theor. Phys. 128 (2012), 955-970

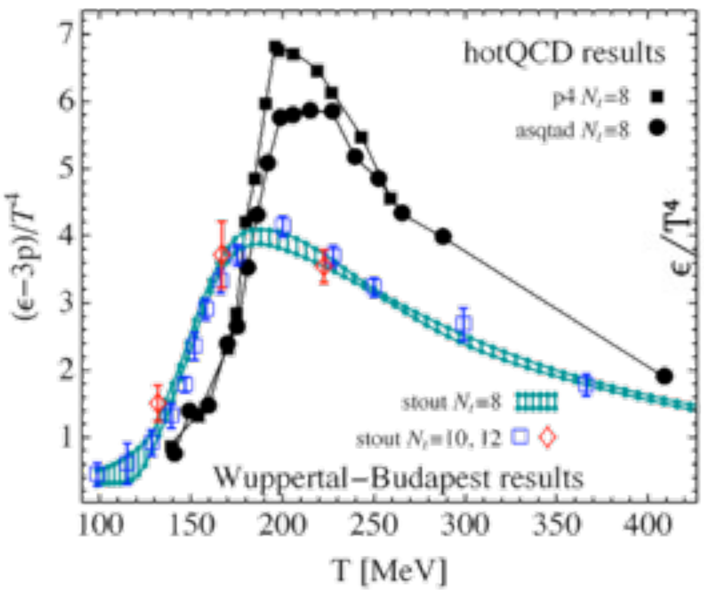
$$T = 1/N_t a_t$$

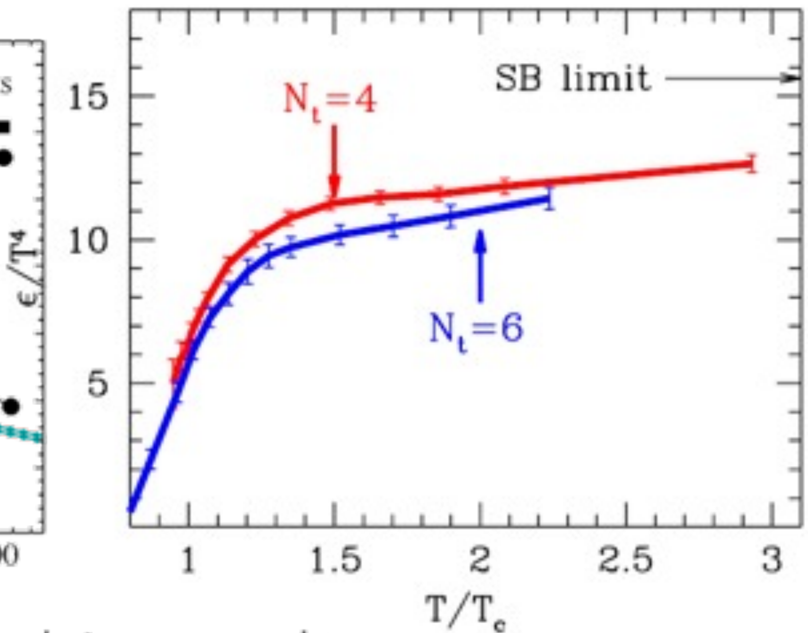
$$a_t \rightarrow 0 \text{ (Continuum Limit)} \qquad N_t \rightarrow \infty$$

Y.Aoki et al.,

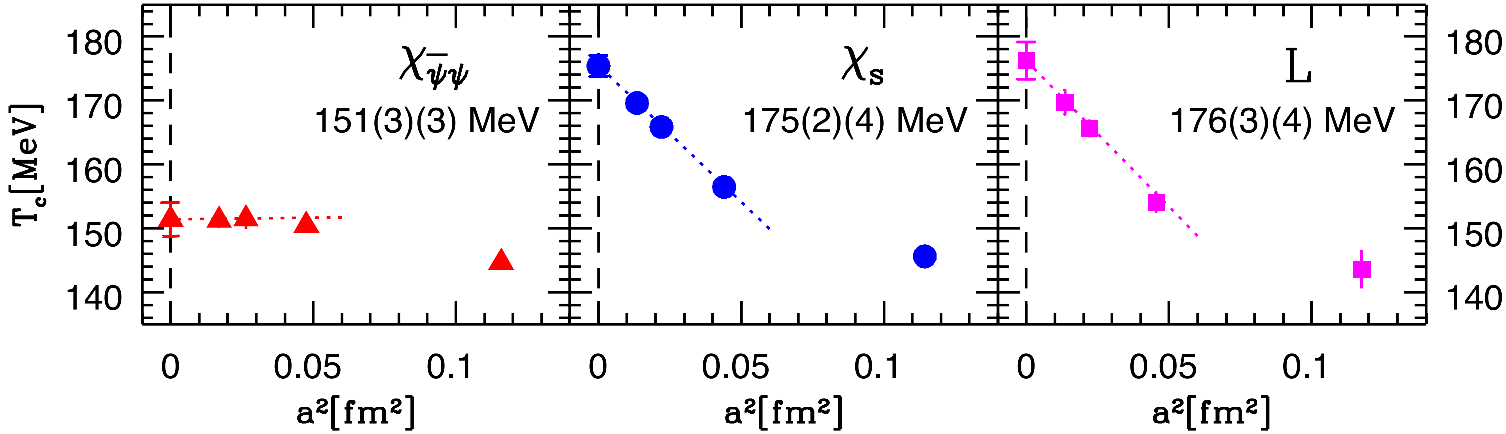
hep-lat/0510084

Lattice QCD Thermodynamics





g. 10. The normalized trace anomaly obtained in our study is compared to recent sults from the HotQCD Collaboration [24, 25].



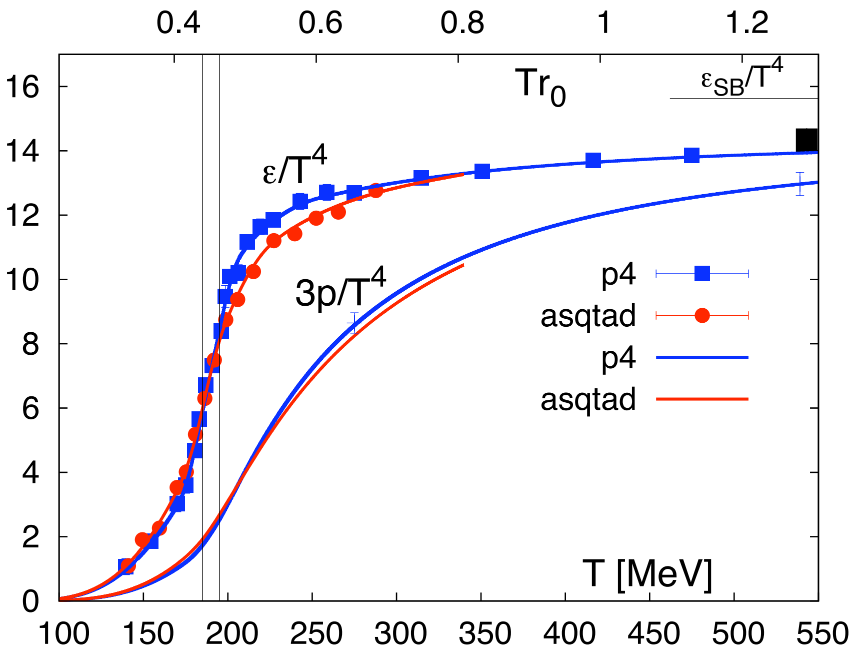
Z. Fodor, S.D. Katz arXiv:0908.3341v1

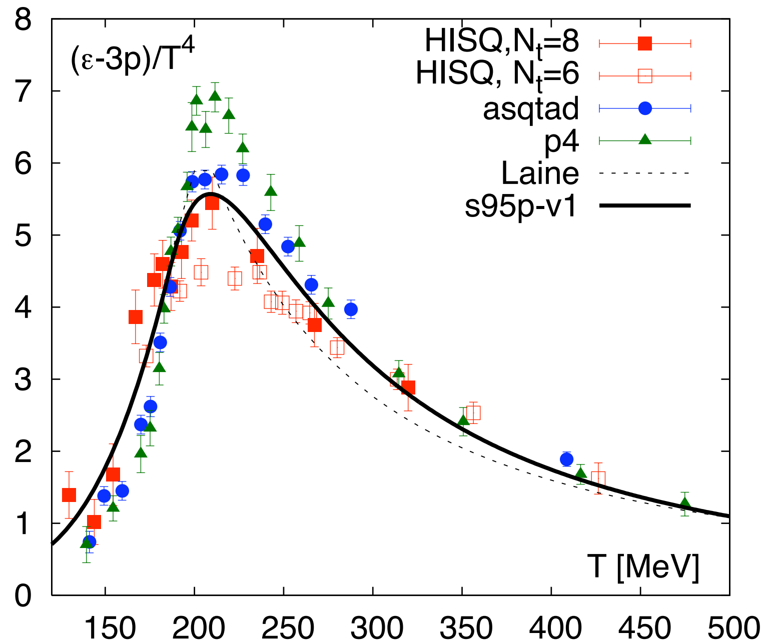
EoS by Lattice

Hungary-Wuppertal

Oh, Great Fighting ！
金と人の勝負だな

US-Bielefeld





Bazavov et al. (HotQCD)
arXiv 0903.4379

Bazavov and Petreczky (HotQCD)
arXiv:1005.1131

T_c summary of the Wuppertal-Budapest group

list of pseudocritical temperatures (various observables)

	$\chi_{\bar{\psi}\psi}/T^4$	$\Delta_{l,s}$	$\langle\bar{\psi}\psi\rangle_R$	χ_2^s/T^2	ϵ/T^4	$(\epsilon$-3p)/T^4
WB'10	147(2)(3)	157(3)(3)	155(3)(3)	165(5)(3)	157(4)(3)	154(4)(3)
WB'09	146(2)(3)	155(2)(3)	-	169(3)(3)	-	-
WB'06	151(3)(3)	-	-	175(2)(4)	-	-

all numbers (in a given coloumn) are in complete agreement
different variables give different pseudocritical T_c-s: 147–165 MeV
reason: the transition is a broad one with 30-40 MeV broadness

3% shift to lower values between 2006 and 2009
reason: 3% experimental change in f_K (no change in lattice results)

Fodor, ISMD2010

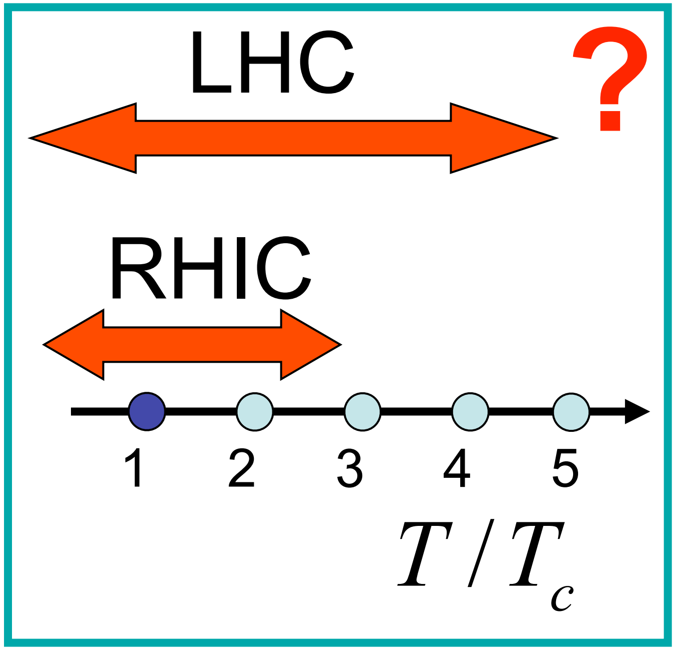
13年6月26日水曜日

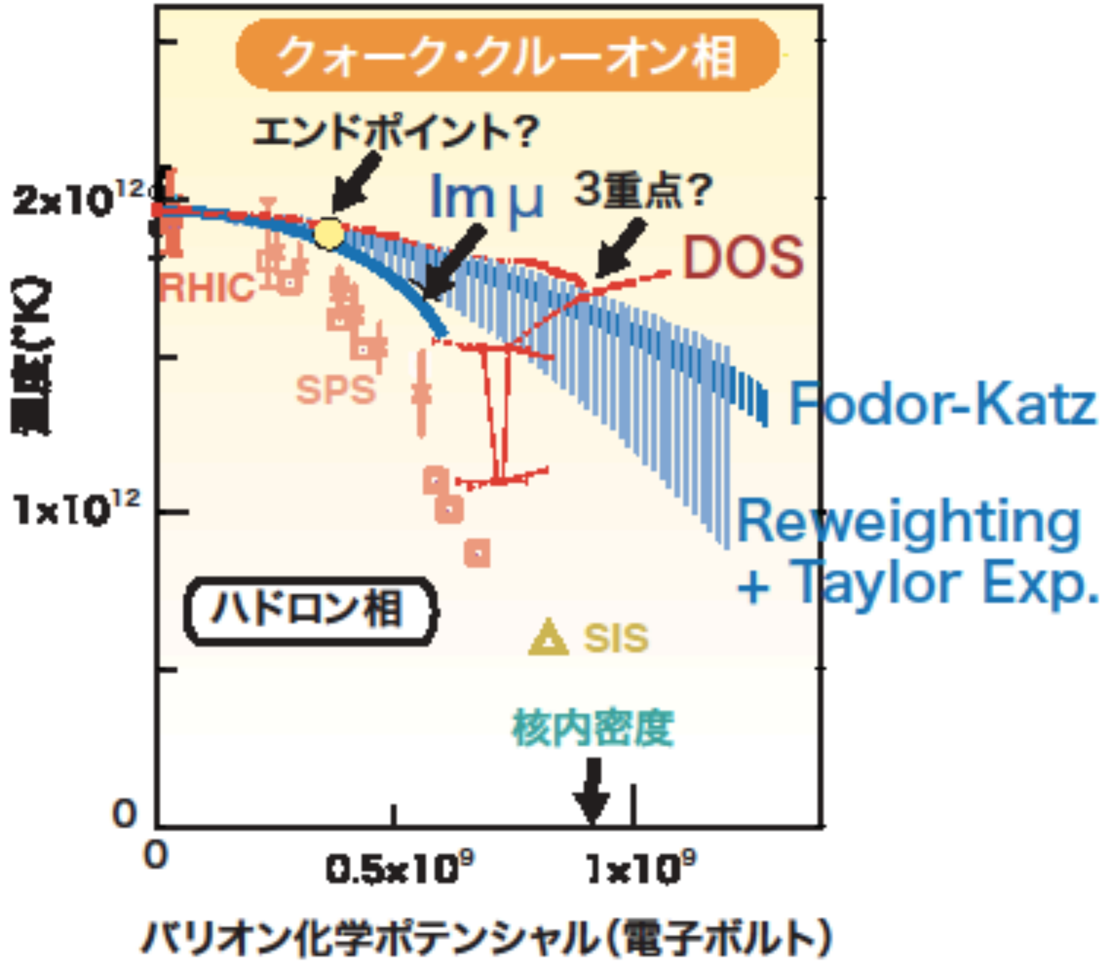
A brief history of the T_{pc} "controversy"

- MILC (2005) 169(12)(4) MeV (physical point)
- Cheng *et al.* (2006) 192(7)(4) MeV (physical point)
- HotQCD (2009 paper) did not quote a number for T_{pc} at the physical point.
- Budapest-Wuppertal (2009/10) 147(2)(3) or 165(5)(3) (physical point)
- HotQCD (Lattice 2010 Preliminary) 164(6) MeV (2010) (physical point)

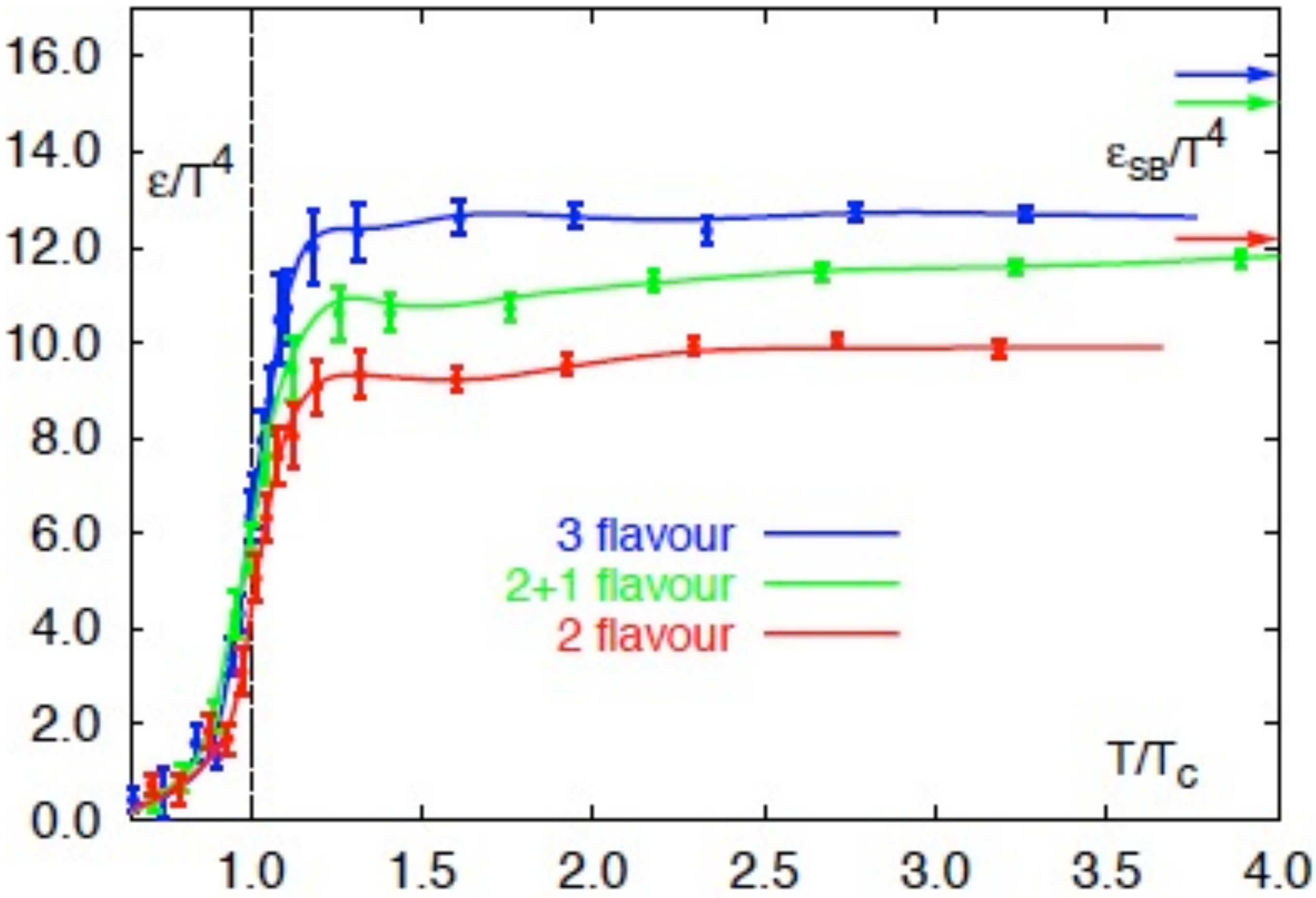
deTar, Atagawa2010

高エネルギー重イオン反応実験と格子QCD





Lattice QCD Calculations
F. Karsch, Lect. Notes Phys. 583 (2002) 209.



状態方程式やPolyakov Lineはだいたい
終わった

しかし、それだけではそこでのダイナミックスを
本当に理解するには不十分

Calculation of Color Dependent Objects -

Color Dependent Potentials

$$3 \times 3^* = 1 + 8$$

In early days, we measured the "Color-Averaged" Potential, although the color-singlet formulation was given by McLerran and Svetitsky

Now we can measure "Color-Singlet" Potential.

Polyakov Loop Correlations

- McLerran and Svetisky, Phys.Rev.D24(1981)450
- “Static” quark

$$\left(\frac{1}{i}\frac{\partial}{\partial t} - t^a A_0^a(\vec{x},t)\right)\psi(\vec{x},t) = 0$$

$$\Rightarrow \quad \psi(\vec{x},t) = T\exp\left(i\int_0^t dt' t^a A_0^a(\vec{x},t')\right)\psi(\vec{x},0)$$

$$\Rightarrow \quad L(\vec{x})\psi(\vec{x},0)$$

$$L(\vec{x}) \equiv U_t(\vec{x},N_t)U_t(\vec{x},N_t-1)\cdots U_t(\vec{x},1)$$

$\mathrm{Tr}L(\vec{x})$:Polyakov Line

$\bar{q}q$ state

$$e^{-\beta F_{q\bar{q}}} \propto \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$|\phi\rangle = \psi^a(\vec{x},0)^\dagger (\psi^c)^b(\vec{x},0)^\dagger |Gluons\rangle$$

a,b: Color indices ψ^c : anti-quark

$$e^{-\beta F_{q\bar{q}}} \propto \sum_{a,b,Gluons} \langle Gluons|\psi^a(\vec{x}_1,0)(\psi^c)^b(\vec{x}_2,0)$$

$$\times e^{-\beta H} \psi^a(\vec{x}_1,0)^\dagger (\psi^c)^b(\vec{x}_2,0)^\dagger |Gluons\rangle$$

$$= \sum_{a,b,Gluons} \langle Gluons| \, e^{-\beta H} \psi^a(\vec{x}_1,\beta)\psi^a(\vec{x}_1,0)^\dagger$$

$$\times (\psi^c)^b(\vec{x}_2,\beta)(\psi^c)^b(\vec{x}_2,0)^\dagger |Gluons\rangle$$

$$= \sum_{a,b,Gluons} \langle Gluons|\, e^{-\beta H}\, L(\vec{x}_1)^{aa'}\, \psi^{a'}(\vec{x}_1, 0)$$

$$\times \psi^{a}(\vec{x}_1, 0)^{\dagger} L(\vec{x}_2)^{\dagger bb'} (\psi^{c})^{b'}(\vec{x}_2, 0)(\psi^{c})^{b}(\vec{x}_2, 0)^{\dagger} \,|\, Gluons >$$

$$= \sum_{gluons} < Gluons \,|\, e^{-\beta H} Tr L(\vec{x}_1) Tr L(\vec{x}_2) \,|\, Gluons >$$

$$\propto < Tr L(\vec{x}_1) Tr L^{\dagger}(\vec{x}_2) >$$ Color averaged

Here we used $[\psi^{a}(\vec{x}, 0), \psi^{b}(\vec{x}', 0)^{\dagger}] = \delta_{a,b}\delta_{\vec{x},\vec{x}'}$

and similar relation for anti-quark fields.

Color singlet $\bar{q}q$

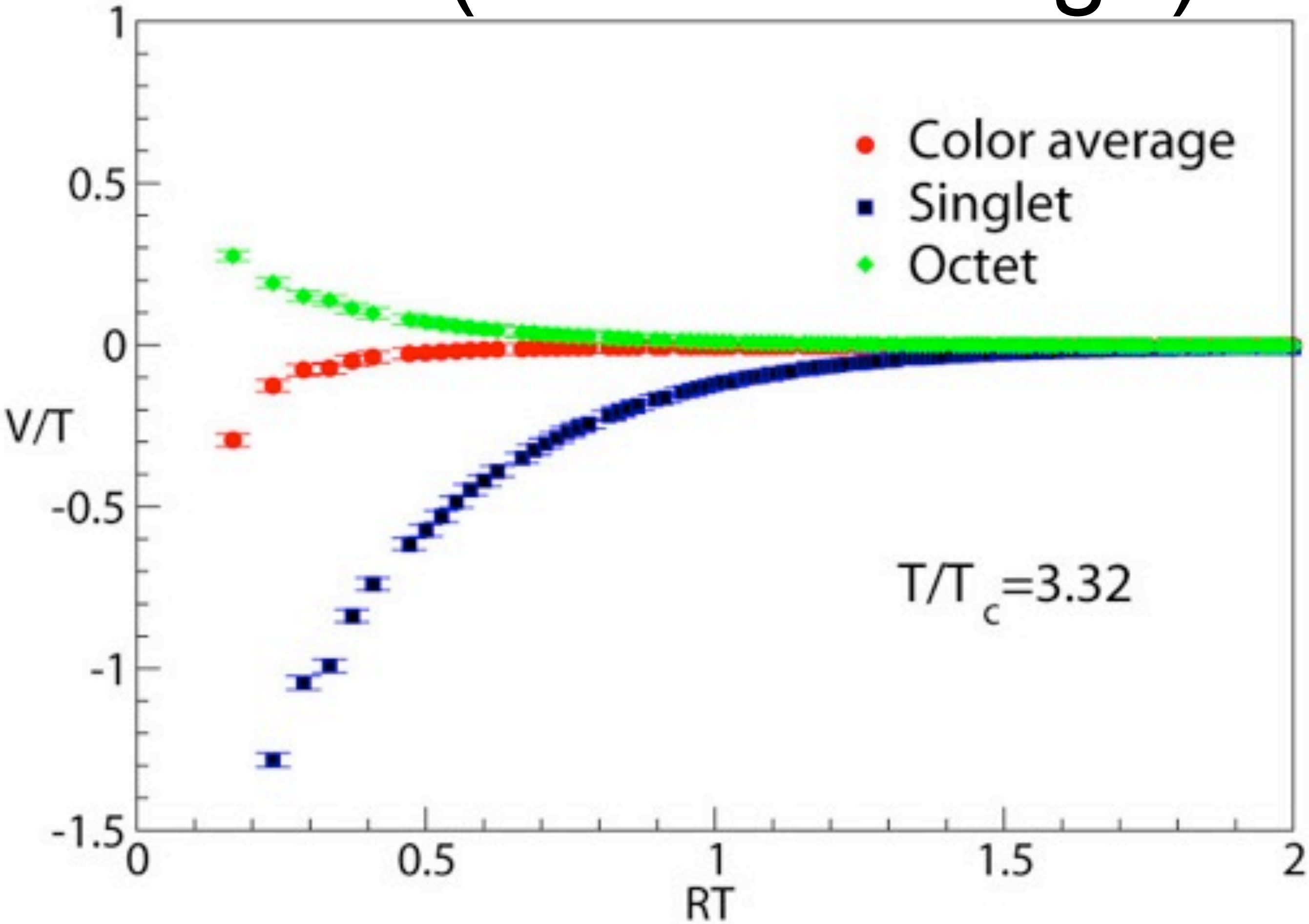
- 3×3* = 1+8

$$e^{-\beta F_1} = \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$|\phi> = \sum_{a} \psi^{a}(\vec{x}_1, 0)^{\dagger} (\psi^{c})^{a}(\vec{x}_2, 0)^{\dagger} | Gluons >$$

$$\Rightarrow \quad e^{-\beta F_1} : < Tr L(\vec{x}_1) L^{\dagger}(\vec{x}_2) >$$

Color-dependent Potentials (Landau Gauge)



$$24^3 \times 6$$

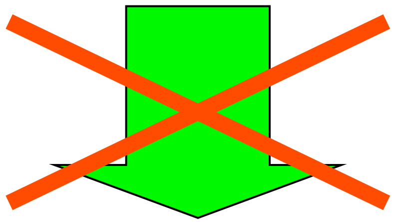
Quench

T.Saito and A.Nakamura. Prog. Theor. Phys. Vol. 111 No. 5 (2004) pp. 733-743

See also Maezawa et al (WHOT-QCD Collaboration) Prog. Theor. Phys. 128 (2012), 955–970

Deconfinement

(Disappearing of the confinement potential)

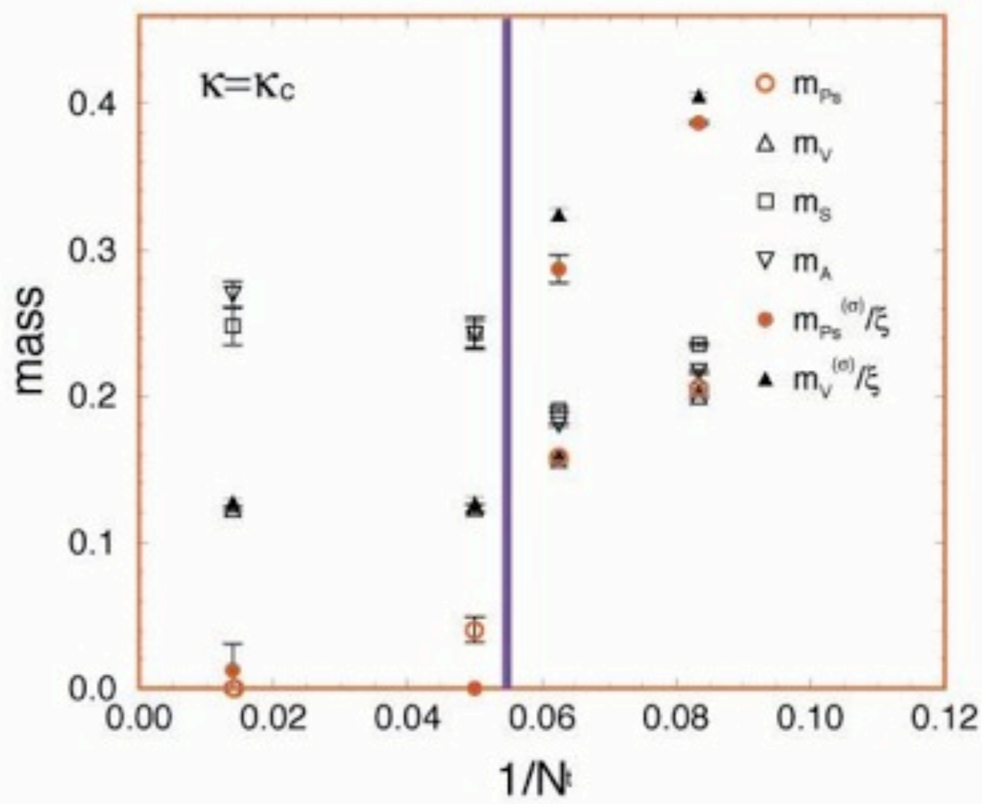


No Bound State

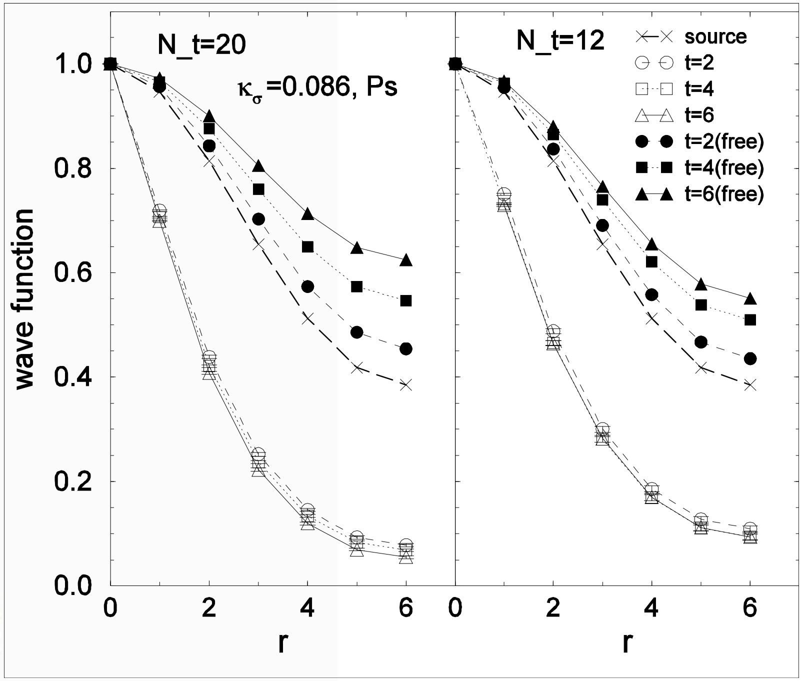
- QED is a Deconfinement theory, but there are Positroniums.
- Mass and Width may change.

- Hadrons at finite Temperature -

QCD-Taro Collaboration, Phys.Rev. D63 (2001) 054501,hep-lat/0008005

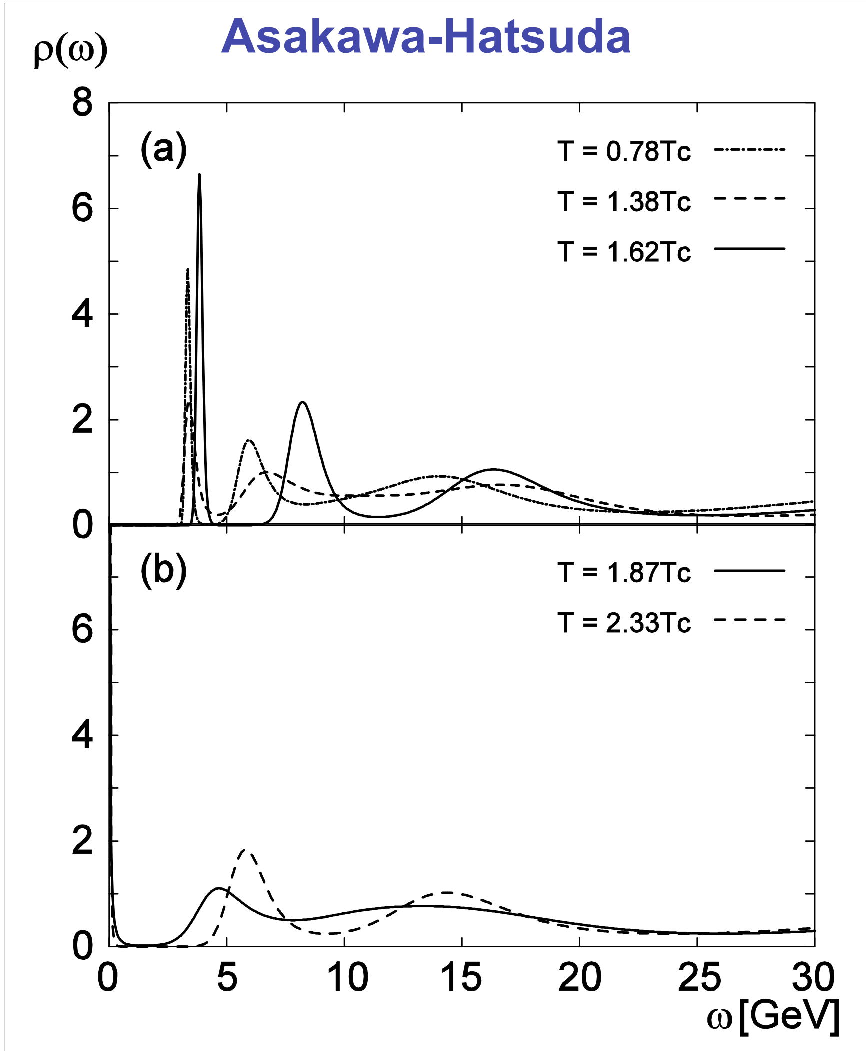


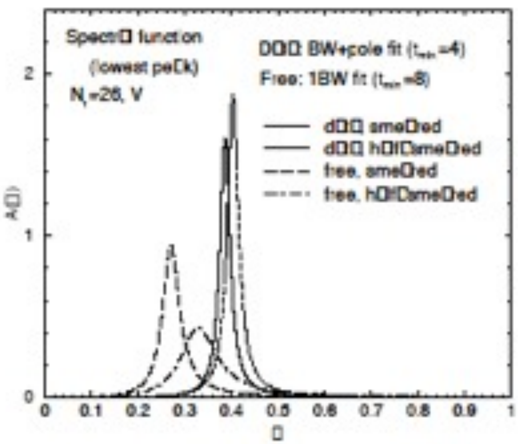
Confined Deconfined



Spectral Functions at finite *T*

- Asakawa-Hatsuda
  - Phys.Rev.Lett. 92 (2004) 012001
- Umeda et al.
  - Nucl.Phys. A721 (2003) 922
- Datta et al.
  - Phys.Rev. D69 (2004) 094507





Umeda et al.

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu, Nucl.Phys.B400(1993)267

$$<< \frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')] >> \equiv \frac{1}{Z}\mathrm{Tr}(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H})$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x}) = e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_\beta^{ret/adv}(t,\vec{x};t',\vec{x}') = \pm\theta(t-t'/t'-t) << \ldots >>$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_\beta^{ret/adv}(\omega,\vec{p})$$

$$K_\beta^{ret/adv}(\omega,\vec{p}) = \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = \langle\langle T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') \rangle\rangle$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

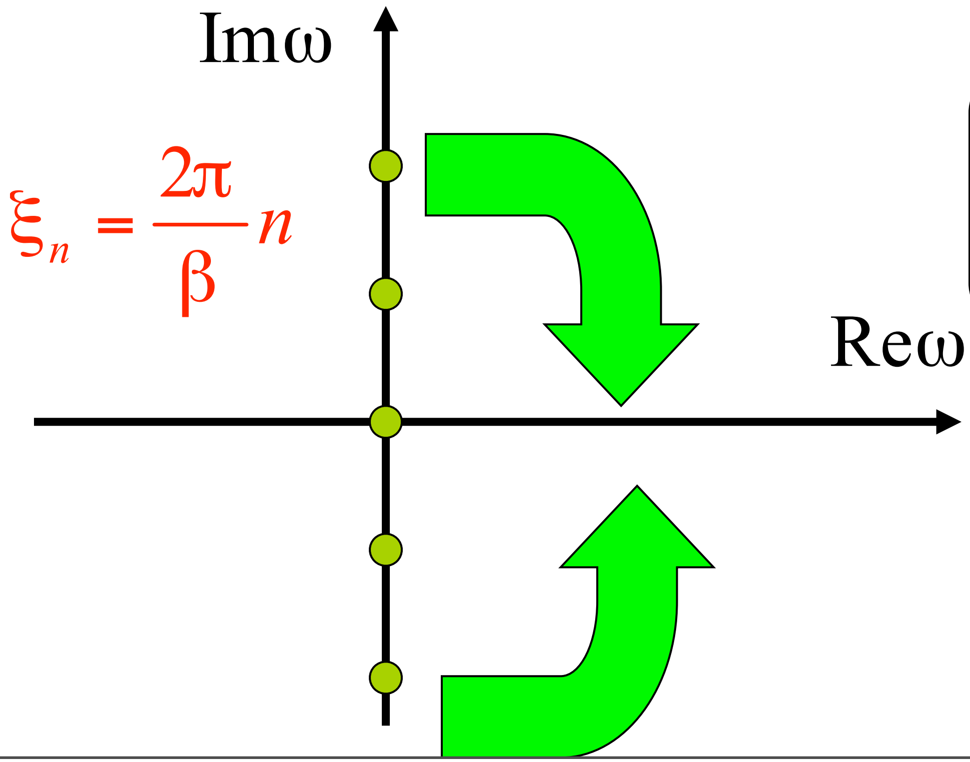
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau\, e^{-i\xi_n(\tau-\tau')} G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0, \pm 1, \pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

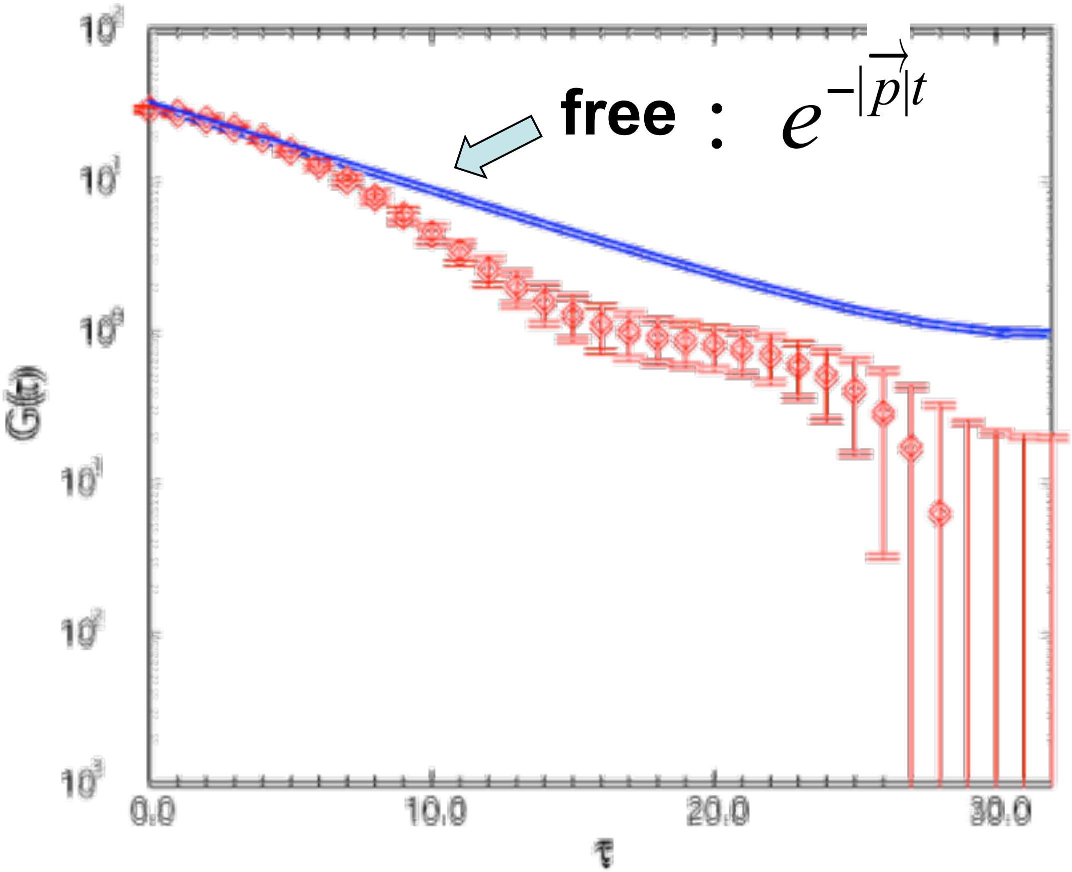
$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



On the lattice, we measure Temperature Green function at $\omega = \xi_n$

We must reconstruct Advance or Retarded Green function.

Gluon Propagator in the confinement (Quench, SU(3), Old Days Calculation)



$$48^3 \times 64$$

$$\beta = 6.8$$

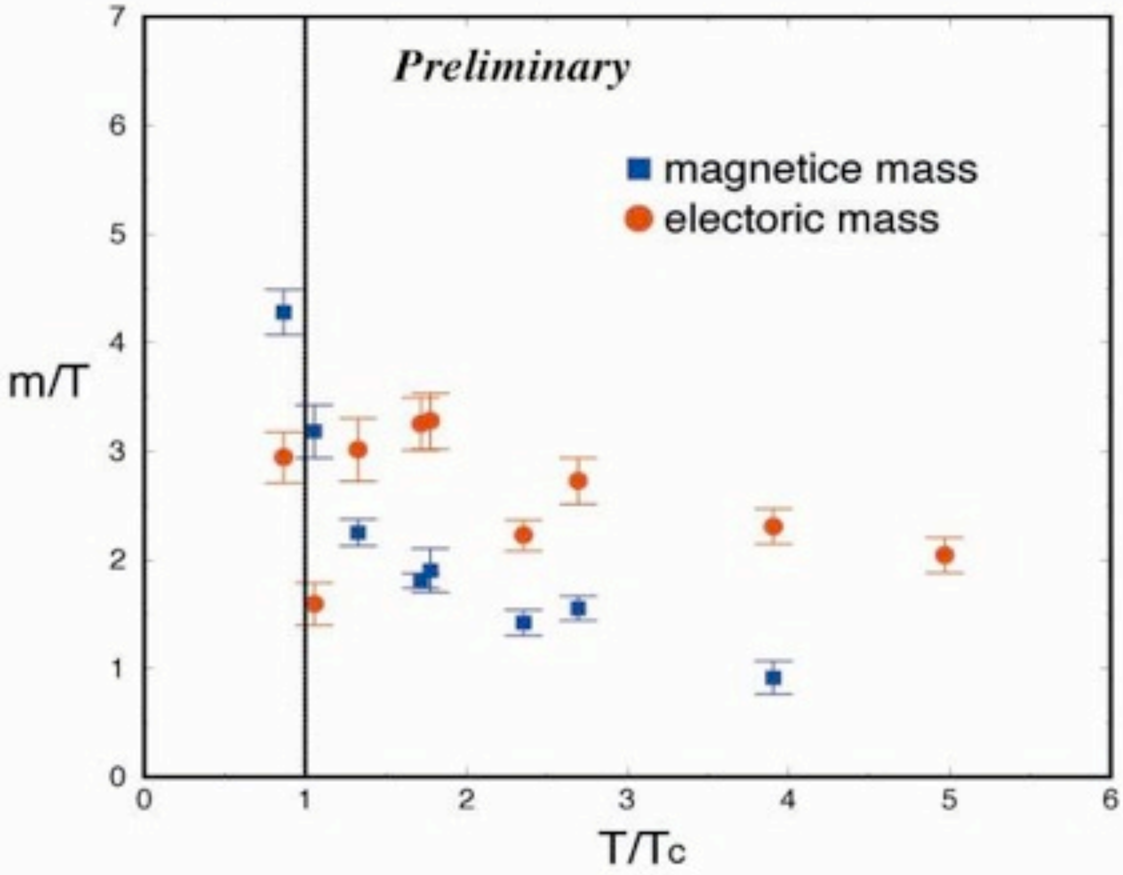
$$\vec{p} = (\frac{2\pi}{N_x}, 0, 0)$$

Landau Gauge

Nakamura, 1995

Gluon’s screening mass

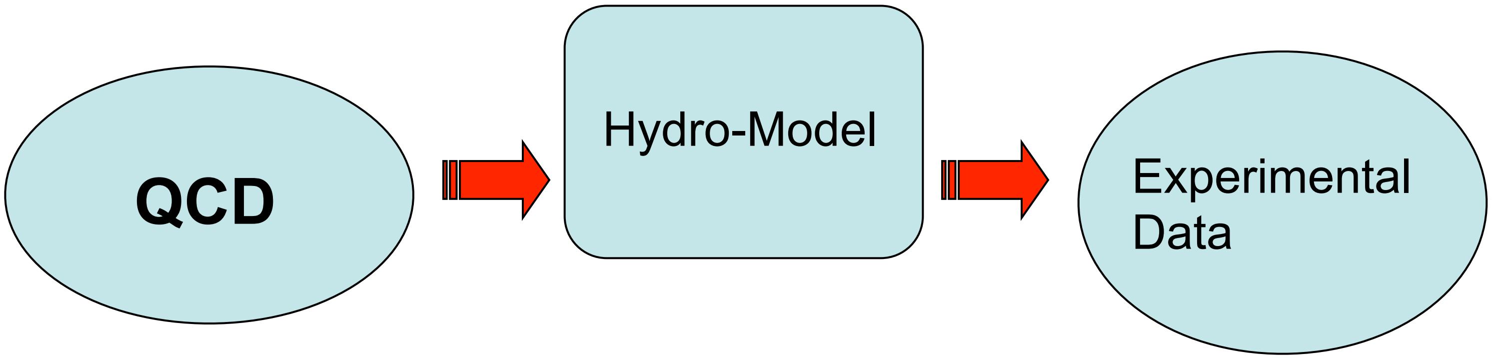
T.Saito hep-lat/0208075



T=Tc～2Tcあたりでは
すごいことになっている？

Transport Coefficients

- A Step towards Gluon Dynamical Behavior.
- They can be (in principle) calculated by a well established formula (Linear Response Theory).
- They are important to understand QGP which is realized in RHIC (and CERN-SPS) and LHC.



Another Personal Motivation

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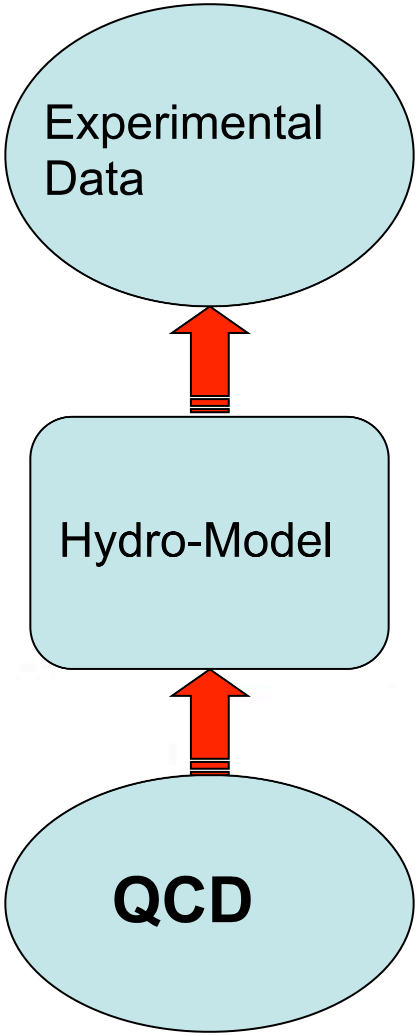
From the Lab came Muroya, Hirano, Nonaka, Morita … who now actively study the hydro-dynamical model.

Yes, I will also study the hydro for supporting young friends.



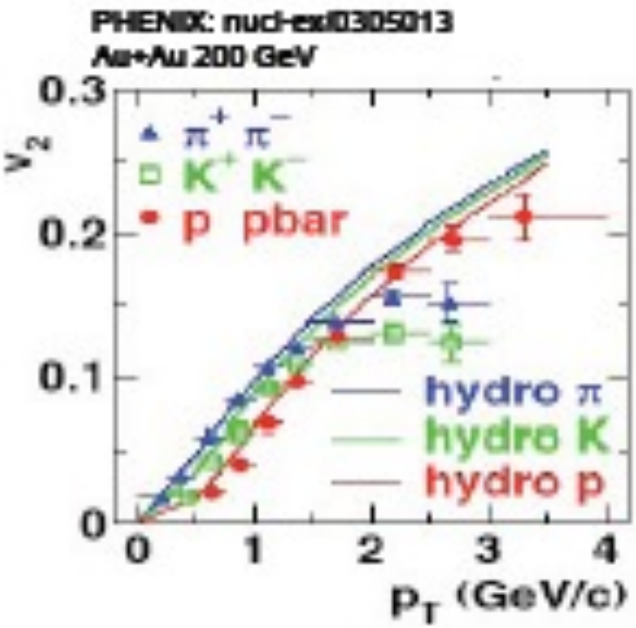
RHIC-data ⇨ Big Surprise !

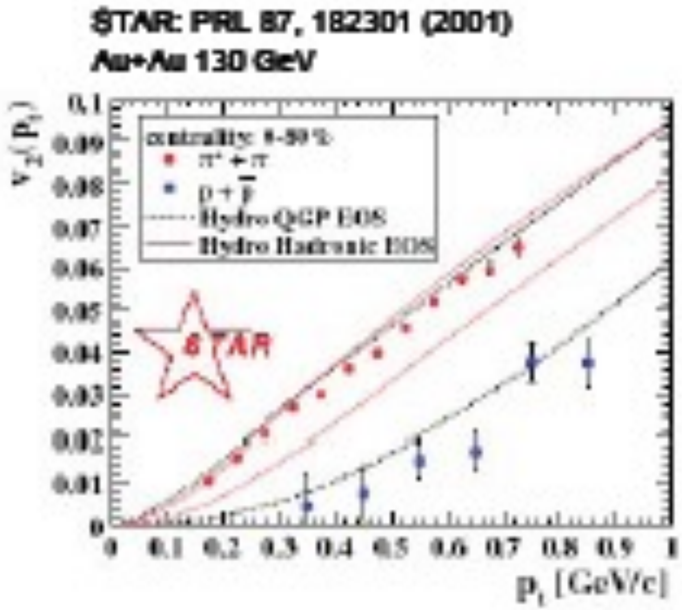
Hydro-dynamical Model describes RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

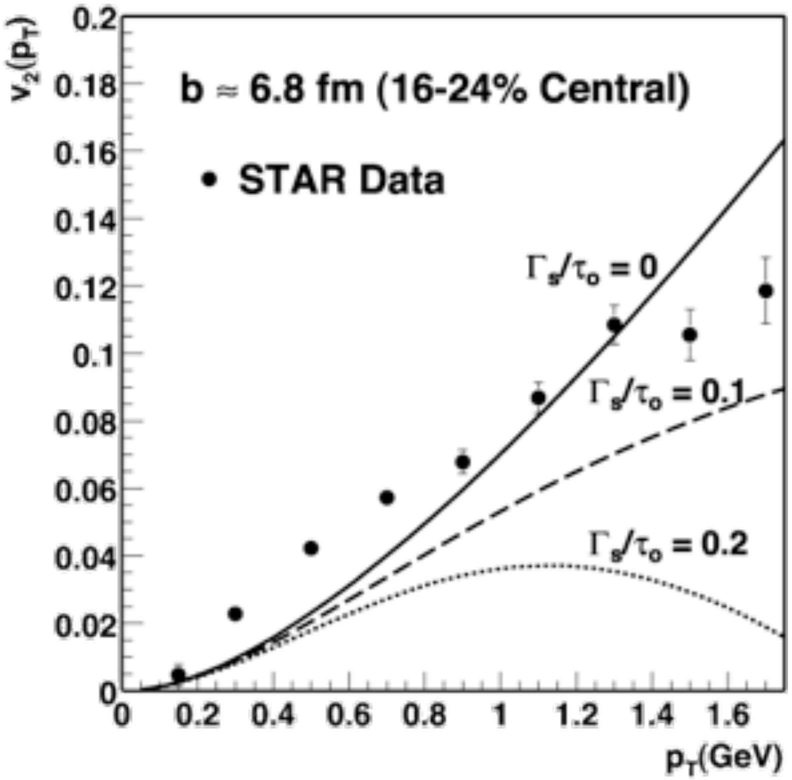
Or not so surprise …

- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen’skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model
    “Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number.”

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Teaney, Phys.Rev. C68 (2003) 034913
(nucl-th/0301099)

$$\Gamma_s \equiv \frac{\frac{4}{3}\eta}{sT}$$



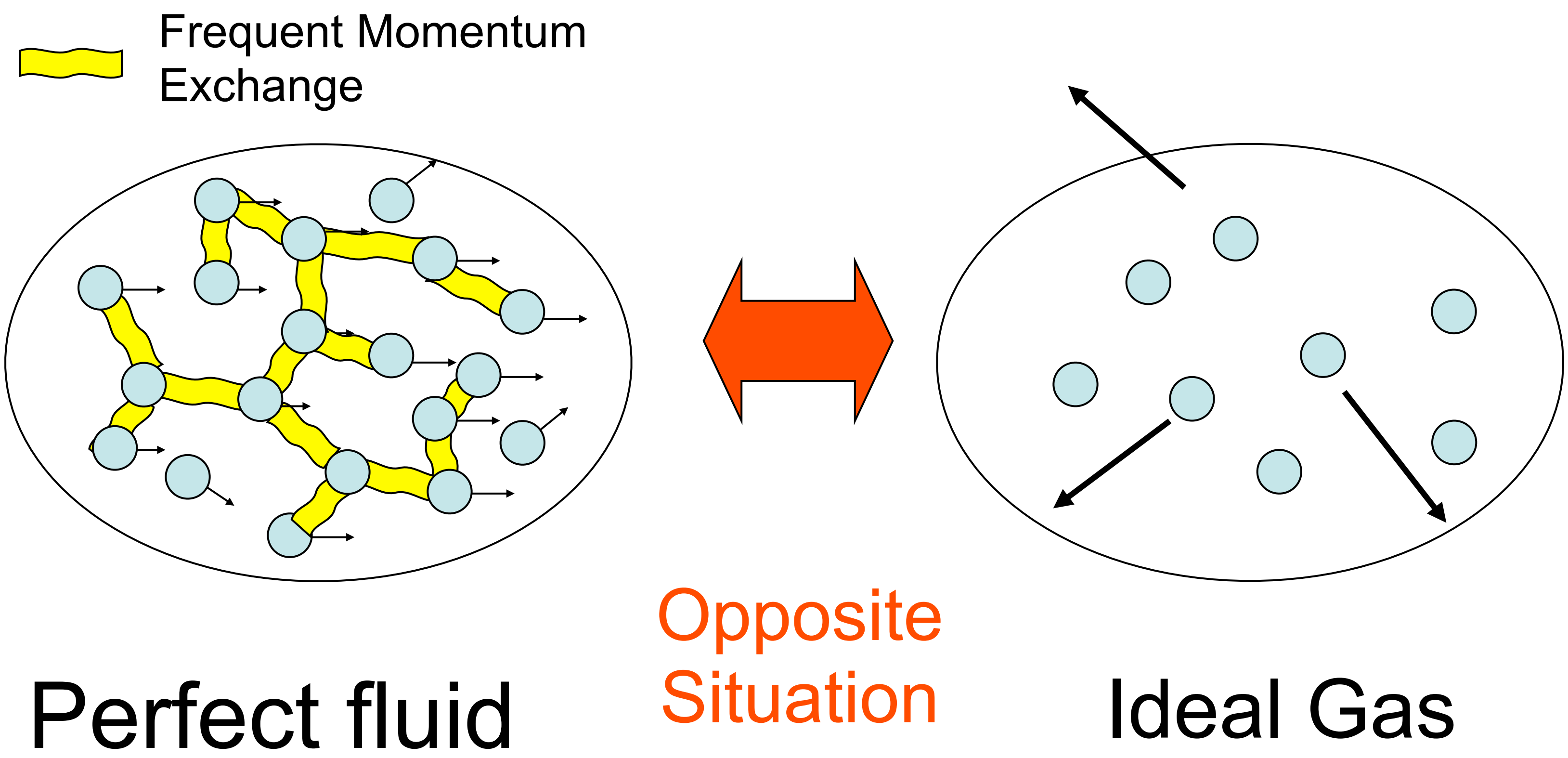
η : shear viscosity

$\tau = \sqrt{t^2 - z^2}$: Time scale of the expansion

Another Big Surprise !

- The Hydrodynamical model assumes zero viscosity,
  i.e., Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?



Literature (1)

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - The first paper to analyze the Hydrodyanamical Model from Field Theory.
  - Applicability Conditions were derived:
    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small

If produced matter at RHIC is (perfect) Fluid, not Free Gas what does it mean ?

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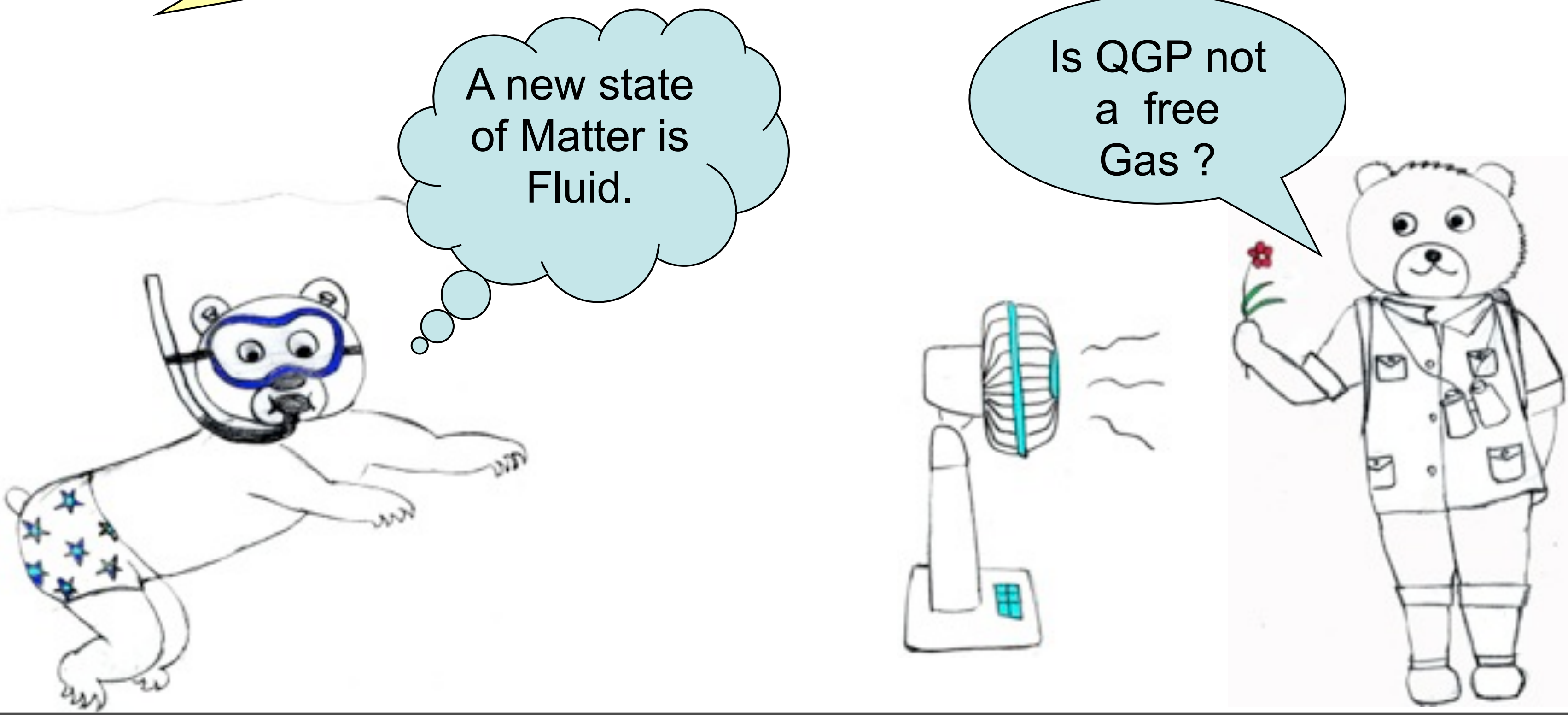


Lowest Perturbation
(Illustration purpose only)

Pressure

$$P = \underbrace{\frac{\pi^2}{90} T^4}_{\text{Ideal Free Gas}} \left(1 - \frac{15}{8} \left(\frac{g}{\pi}\right)^2 + \cdots \right)$$

Viscosity

Lowest Perturbation
(Illustration purpose only)

Pressure

$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8} \left(\frac{g}{\pi} \right)^2 + \cdots \right)$$

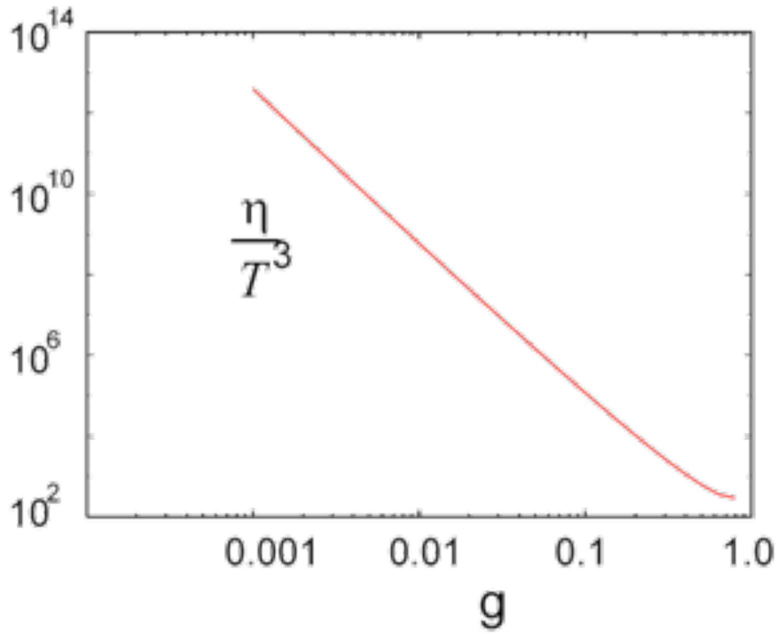
Ideal Free Gas

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126 (N_f = 0), \quad 86.473 (N_f = 2)$$

- At weak coupling, it increases.



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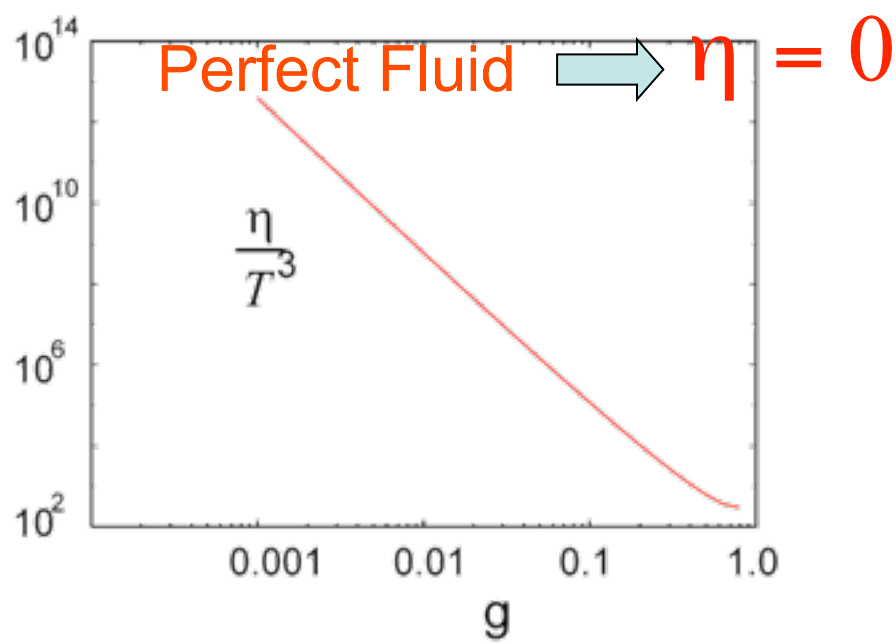


Literature (2)

- G. Baym, H. Monien, C. J. Pethick and D. G. Ravenhall,
  - Phys. Rev. Lett. 16 (1990) 1867.
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0011 (2000) 001, (hep-ph/0010177).
  - Leading-log results"
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0305 (2003) 051, (hep-ph/0302165).
  - Beyond leading log"

Literature (3)

- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
  - Transport Coefficients Formulation
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
  - The first Lattice QCD Calculation
- Aarts and Martinez-Resco, JHEP0204 (2002)053
  - Criticism against the Spectrum Function Ansatz.
- Petreczky and Teaney, hep-ph/0507318
  - Impossible to determine Heavy Quark Transport coefficient

Literature (4)

- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N, Sakai
  Phys.Rev.Lett. 94 (2005) 072305
  hep-lat/0406009

Linear Response Theory

- Zubarev
  "Non-Equilibrium Statistical Thermo-dynamics"
- Kubo, Toda and Saito
  "Statistical Mechanics"

$$\rho : e^{-A+B} : \text{ non-equilibrium statistical operator}$$

$$A = \int d^3x \beta(x,t) u^{\nu} T_{0\nu}(x,t)$$

$$B = \int d^3x \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} T_{\mu\nu}(x,t) \partial^{\mu} (\beta(x,t) u^{\nu})$$

$$\text{Using: } e^{-A+B} = e^{-A} + \int_0^1 d\tau \, e^{A\tau} B e^{-A\tau} e^{-A} + \cdots$$

$$\rho \approx \rho_{eq} + \int_0^1 d\tau \left(e^{A\tau} B e^{-A\tau} e^{-A} - \langle B \rangle_{eq} \right) \rho_{eq}$$

$$\rho_{eq} \equiv e^{-A} / \mathrm{Tr} e^{-A} \rightarrow \exp(-\beta H) / \mathrm{Tr} e^{-A}$$

in the co-moving frame, $u^{\mu} = (1 \quad 0 \quad 0 \quad 0)$

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{eq} +$$

$$+\int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} \partial^{\rho} (\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \langle T_{\rho\sigma}(x',t') \rangle_{eq} \right) \right\rangle_{eq}$$

$$\langle T^{ij} \rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\langle T^{0i} \rangle = -\chi (\beta^{-1}(x,t) \partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\langle p \rangle - \langle p \rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^{i}{}_{i}$$

- One can show

$$(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} = -\beta^{-1}\int_{-\infty}^{t'} dt'' \left\langle T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'') \right\rangle_{ret}$$

Transport Coefficients are expressed by Quantities at Equilibrium

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x}, t) T_{12}(\vec{x}', t') >_{ret}$$

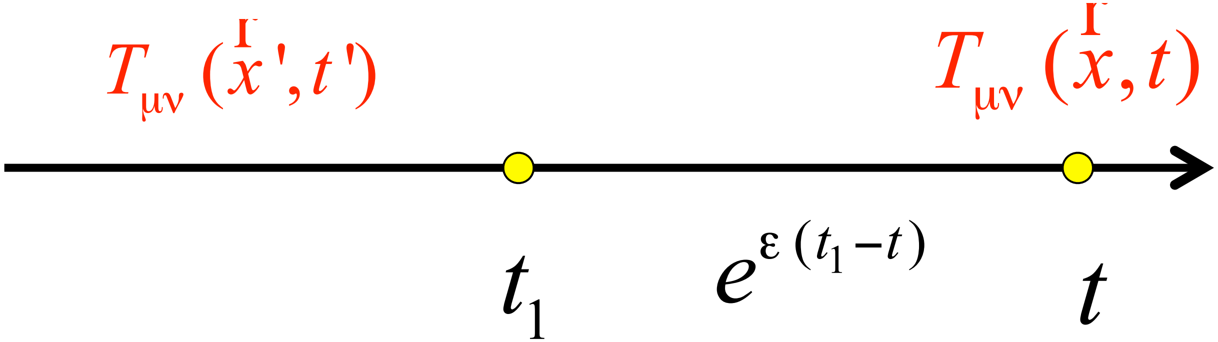
$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x}, t) T_{11}(\vec{x}', t') >$$

$$\chi = -\frac{1}{T} \int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x}, t) T_{01}(\vec{x}', t') >_{ret}$$

η **: Shear Viscosity**

ς **: Bulk Viscosity**

χ **: Heat Conductivity** → we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right) / 2ia^2 g$$

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu,
Nucl.Phys.B400(1993)267

$$<< \frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')] >> \equiv \frac{1}{Z}\mathrm{Tr}\left(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H}\right)$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x}) = e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_\beta^{ret/adv}(t,\vec{x};t',\vec{x}') = \pm\theta(t-t'/t'-t) << \ldots >>$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_\beta^{ret/adv}(\omega,\vec{p})$$

$$K_\beta^{ret/adv}(\omega,\vec{p}) = \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = \langle\langle T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') \rangle\rangle$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

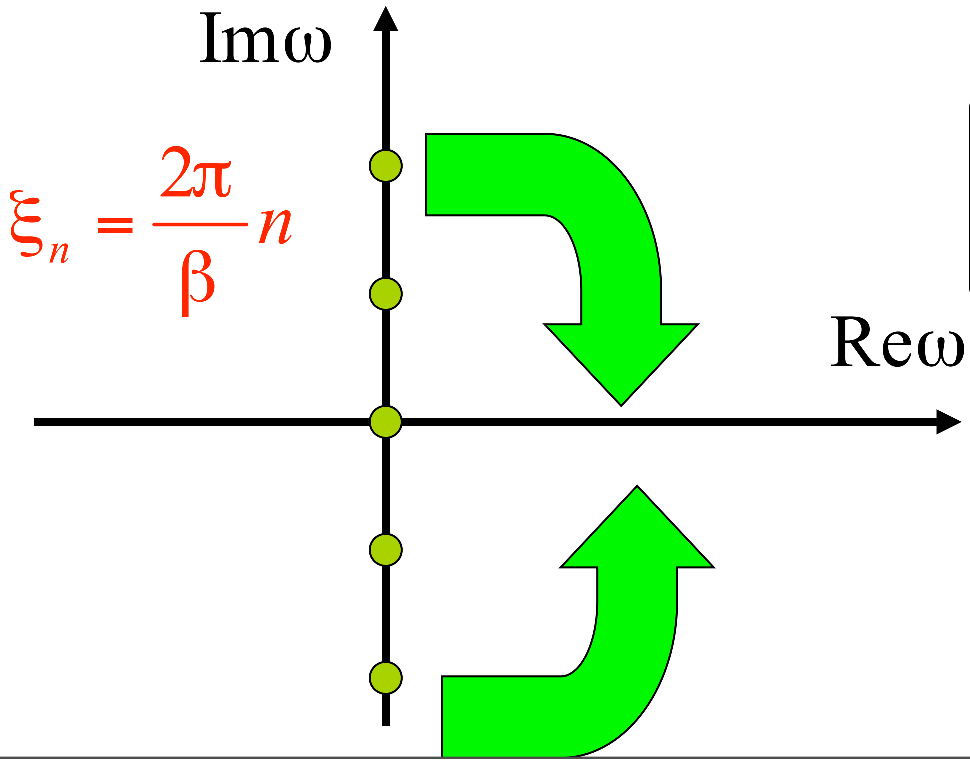
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau\, e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0, \pm 1, \pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



On the lattice, we measure Temperature Green function at $\omega = \xi_n$

We must reconstruct Advance or Retarded Green function.

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$$< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\upsilon}(t,x) T_{\mu\upsilon}(0) >= G_\beta(t,x) = F.T.G_\beta(\omega_n, p)$$

$$G_\beta(p, i\omega_n) = \int d\omega \frac{\rho(p,\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

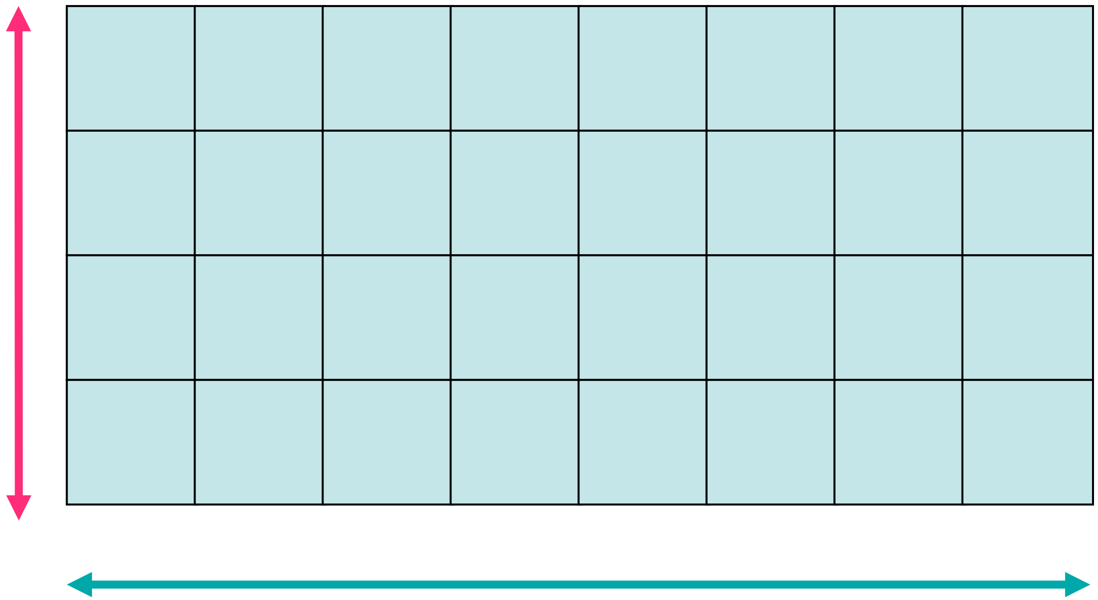
and determine three parameters,

A, m, γ.

We need large Nt !

Some Special Features of Lattice QCD at Finite Temperature

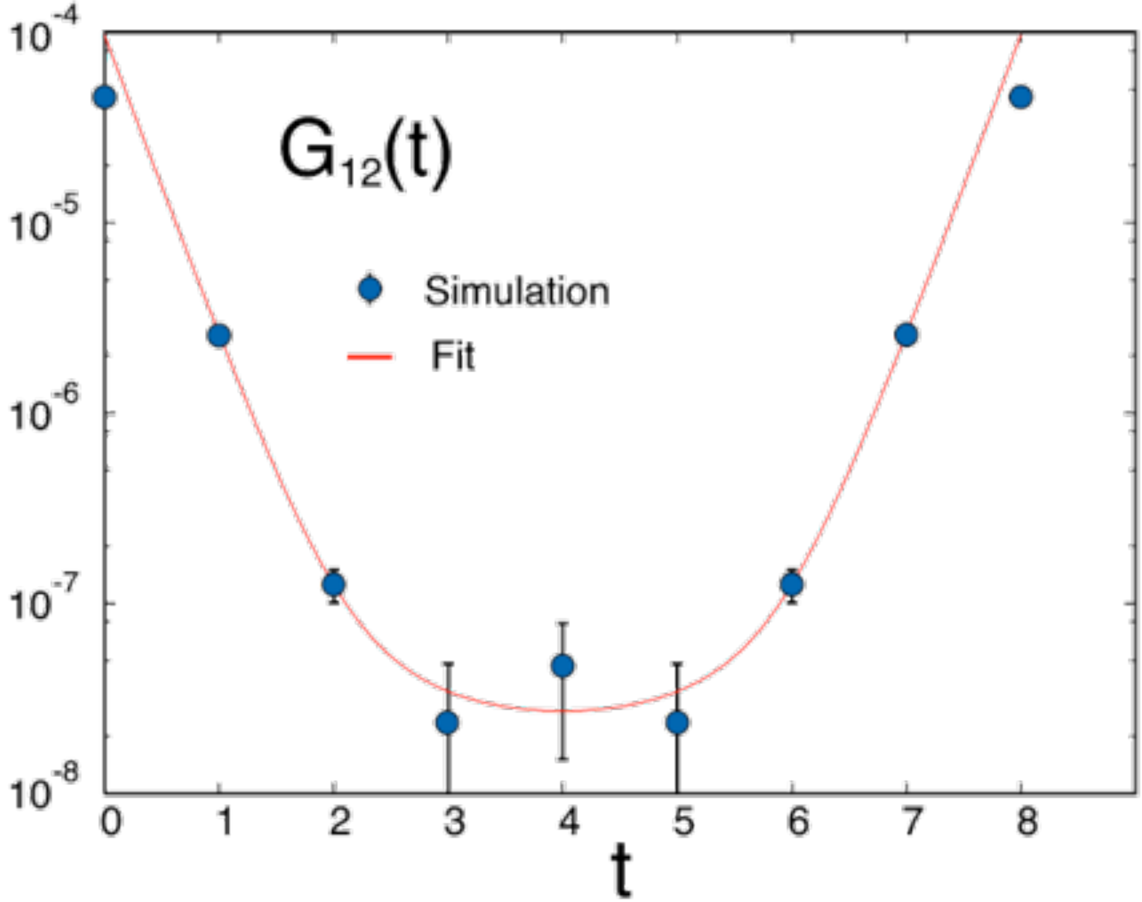
$$N_t a_t = \frac{1}{kT}$$



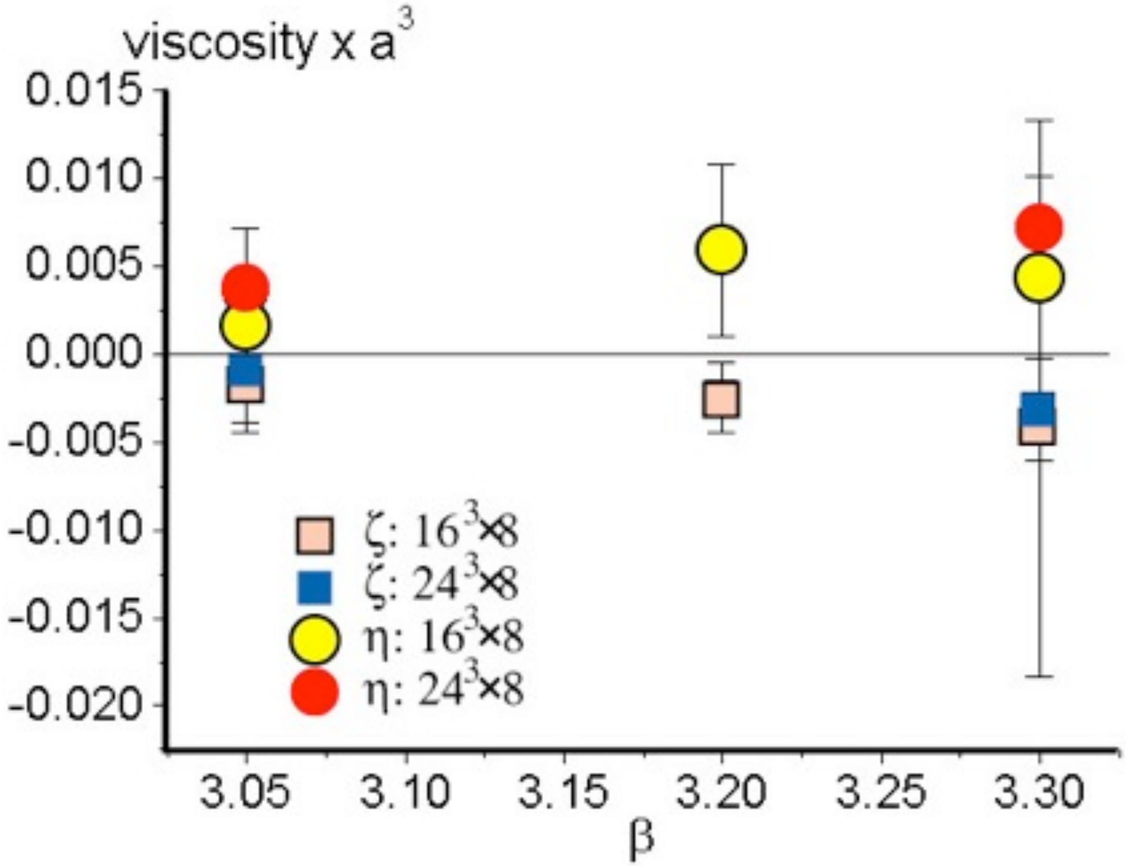
$$N_s a_s$$

High Temperature ⇒ $N_t a_t$: small

Nt=8

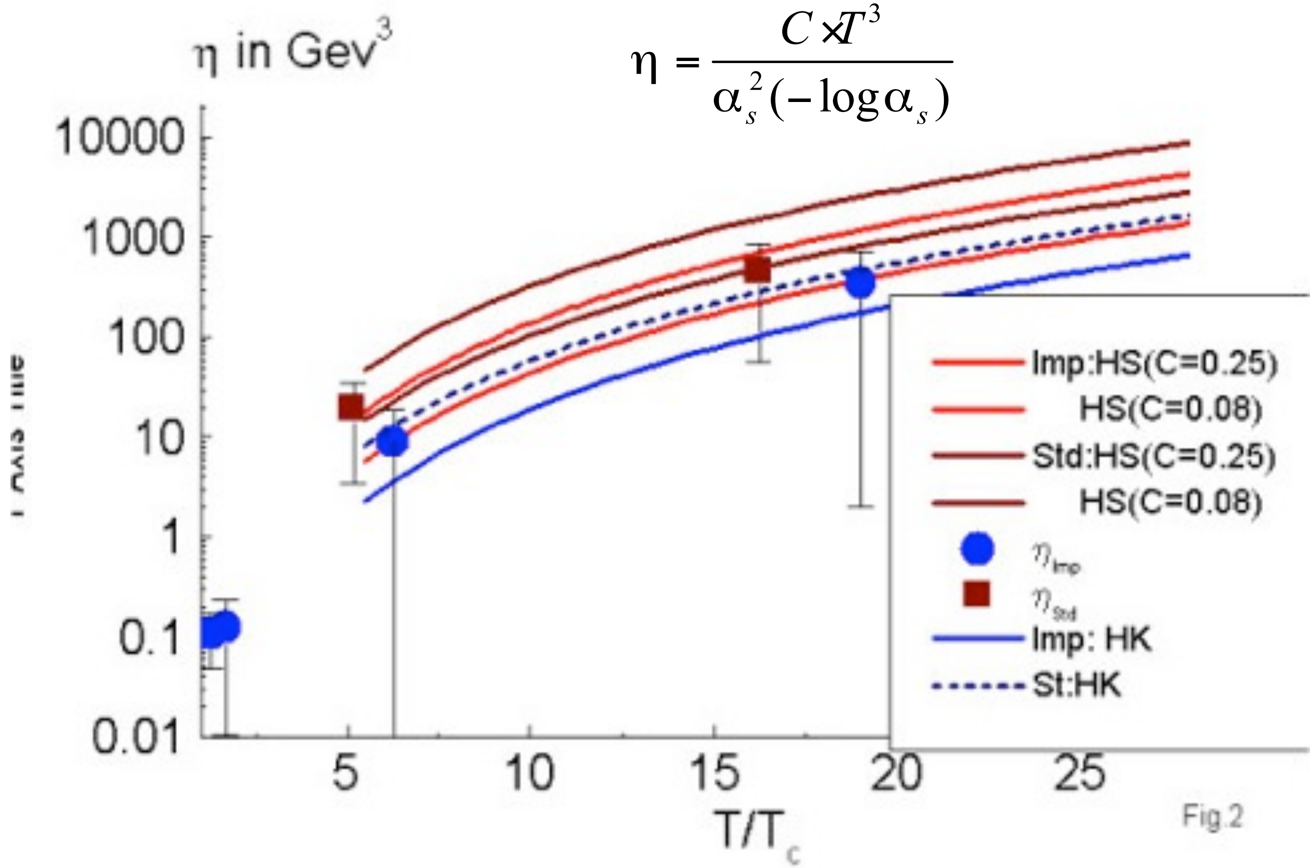


Results: Shear and Bulk Viscosities



Comparison with Pertubative Calculations

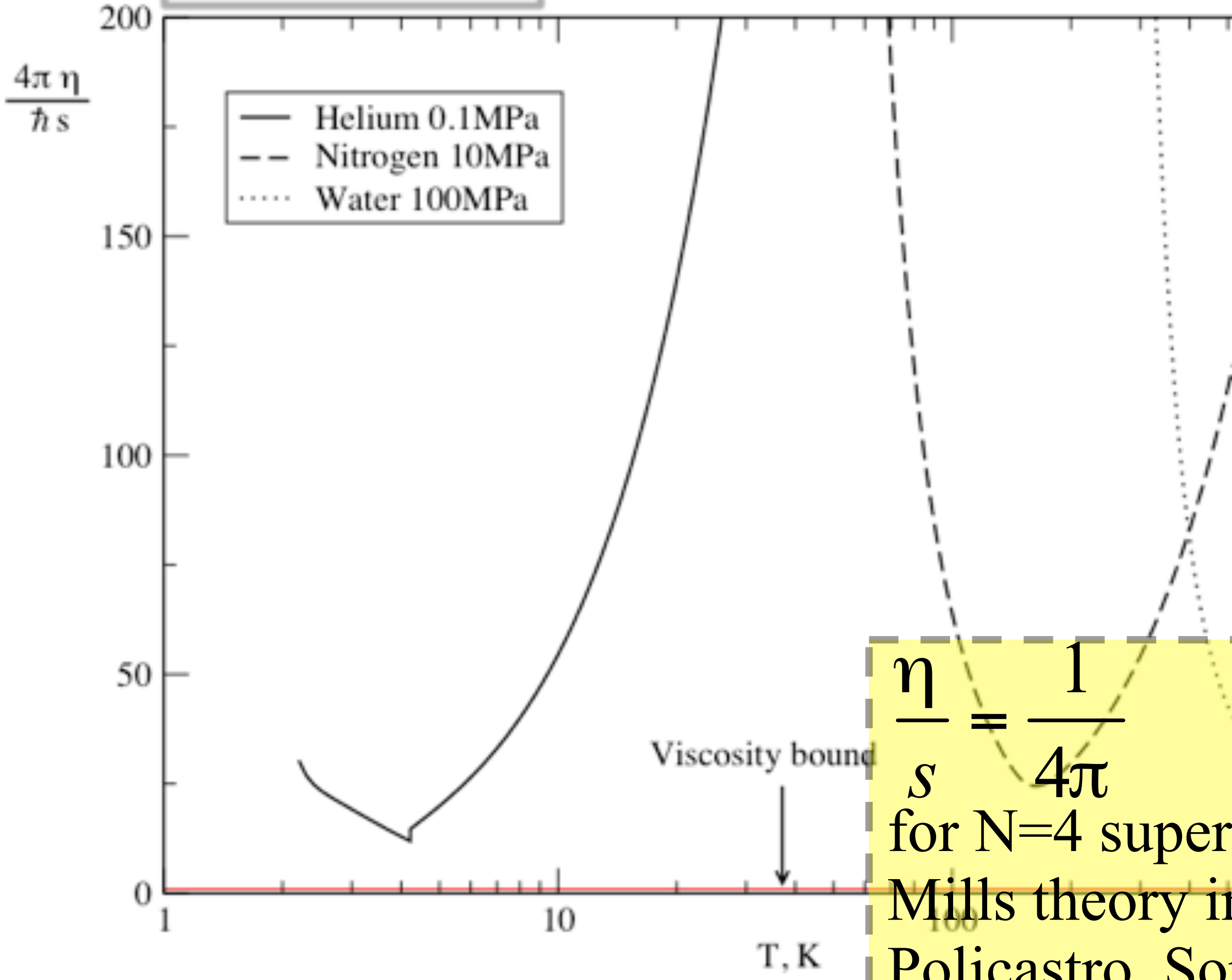
$$\eta = \frac{C \times T^3}{\alpha_s^2(-\log\alpha_s)}$$



Good for T/Tc>5

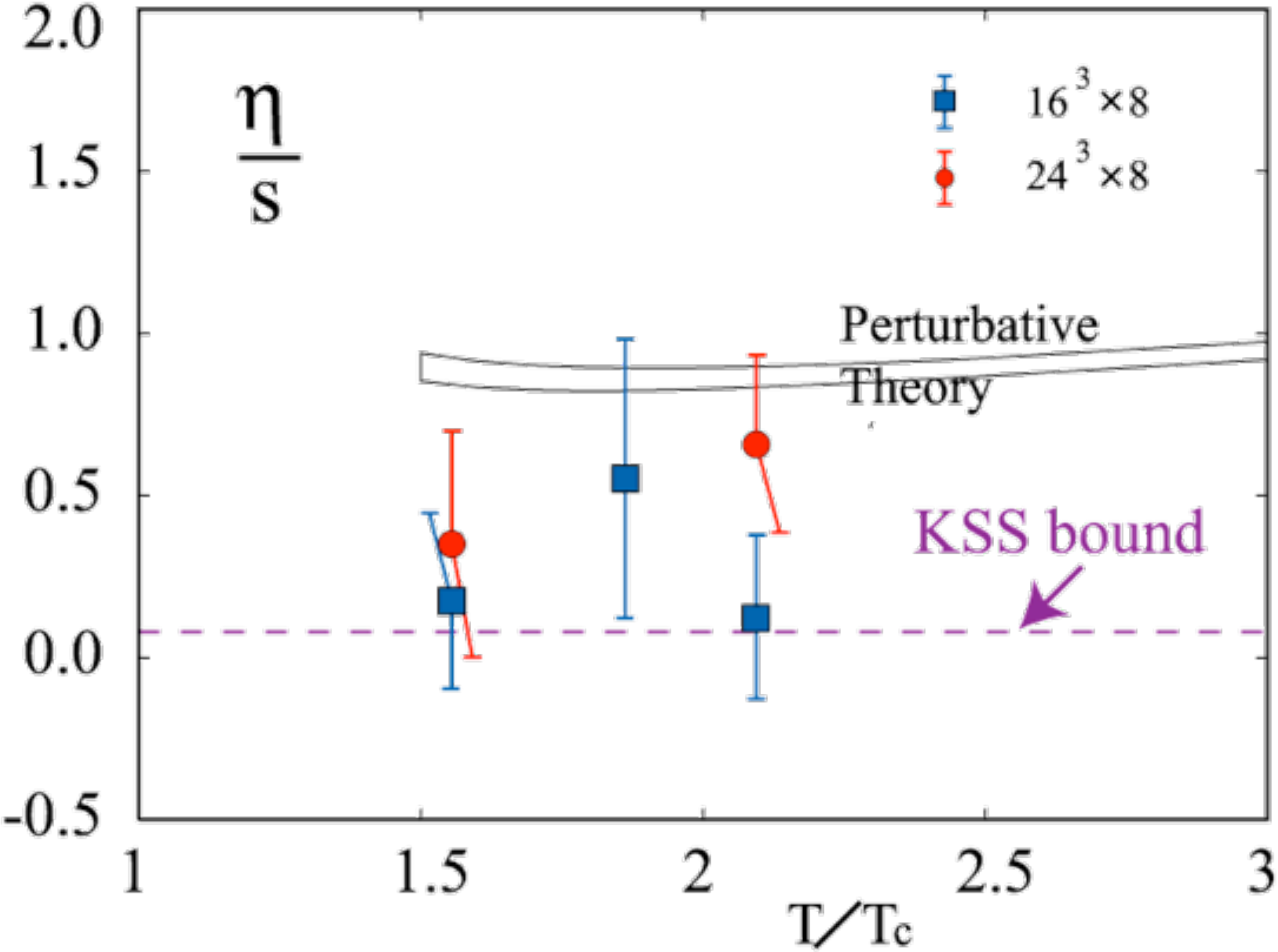
$$\frac{\eta}{s} \geq \frac{1}{4\pi} \; !$$

Kovtun, Son and Starinets, hep-th/0405231



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for N=4 supersymmetric Yang-Mills theory in the large N.
Policastro, Son and Starinets, Phys Rev. Lett. 87 (2001) 081601



2.0
1.5
1.0
0.5
0.0
-0.5
η/s
16^3×8
24^3×8
Perturbative Theory
KSS bound
1
1.5
2
2.5
3
T/Tc

$\frac{\eta}{s}$ can have the lower limit ?

- Counter Example by Prof. Baym
  - We heat up Billiard Balls which have inter-structure. Then Entropy increases. If the surface of the balls does not change, the Viscosity should be the same.

$$\frac{\eta}{s} ® 0$$

- We may give Counter-Argument ?