多重度分布とQCD相転移

中村純

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共同研究者

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松本

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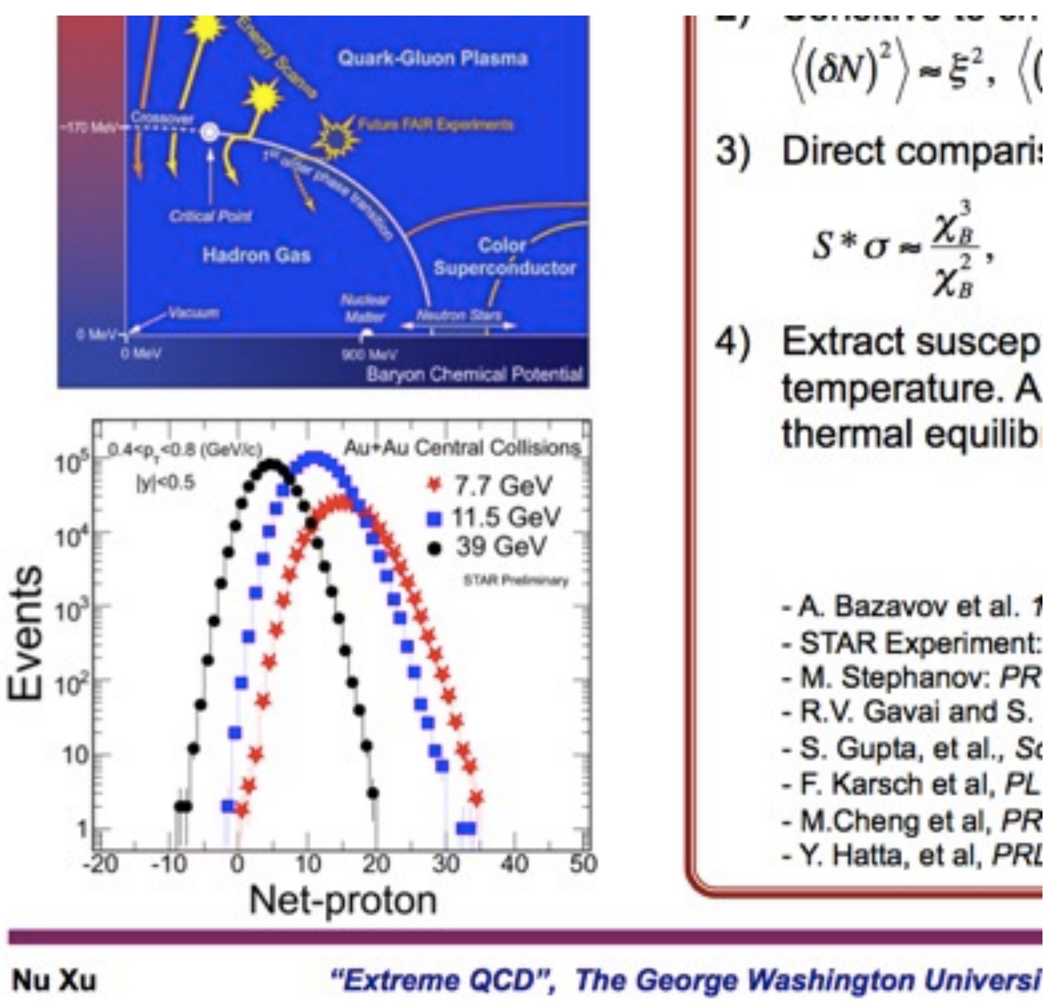
昨日の資料（コー

- http://home.riise.hiroshima-u.ac.jp/~nakamura/panflute/LTKf90v3.tar.gz

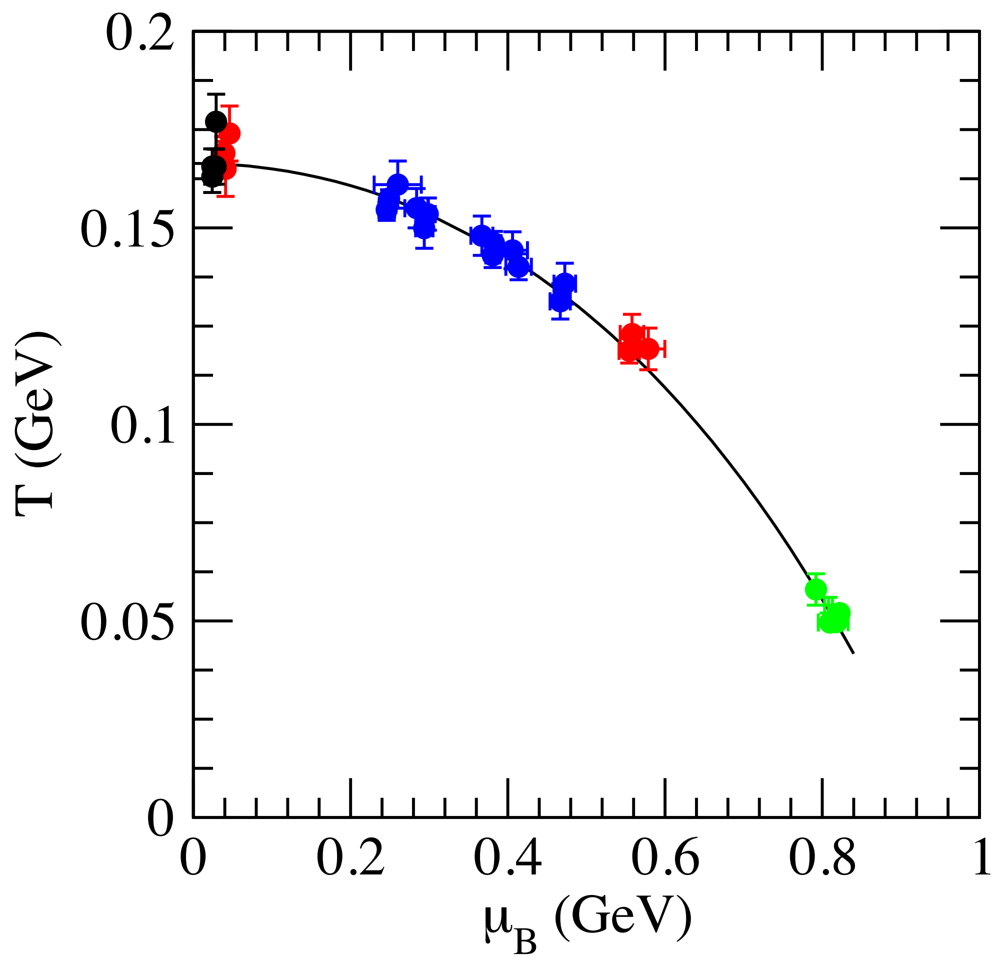
- そもそもは、高エネルギーの現象論をやっていて（博士論文は高エネルギーハドロン原子核反応）、その道具として格子を始めました
- それからxx年

XQCD2012(2012 Aug.ワシントン)で
Nu Xuさんの話を聞きました
QCD Structure 1 (2012 Oct. 武漢)で
Multiplicityの話をたくさん聞きました



Freeze-out

Temperature and Chemical Potential
when the particles were born
were estimated.



Cleymans et al.
Phys.Rev.C73:034905,2006

Canonical Partition Function

⇔

Grand-Canonical Partition Function

$$Z(\xi, T) = \mathrm{Tr}\, e^{-\beta(H-\mu\hat{N})}$$

$$(\beta = 1/T)$$

$$= \sum_n \langle n | e^{-\beta(H-\mu\hat{N})} | n \rangle$$

$$= \sum_n \langle n | e^{-\beta H} | n \rangle (e^{\mu n/T})$$

$$\text{If} \quad [H, \hat{N}] = 0$$

$$= \sum_n \langle n | e^{-\beta H} | n \rangle (e^{\mu n / T})$$

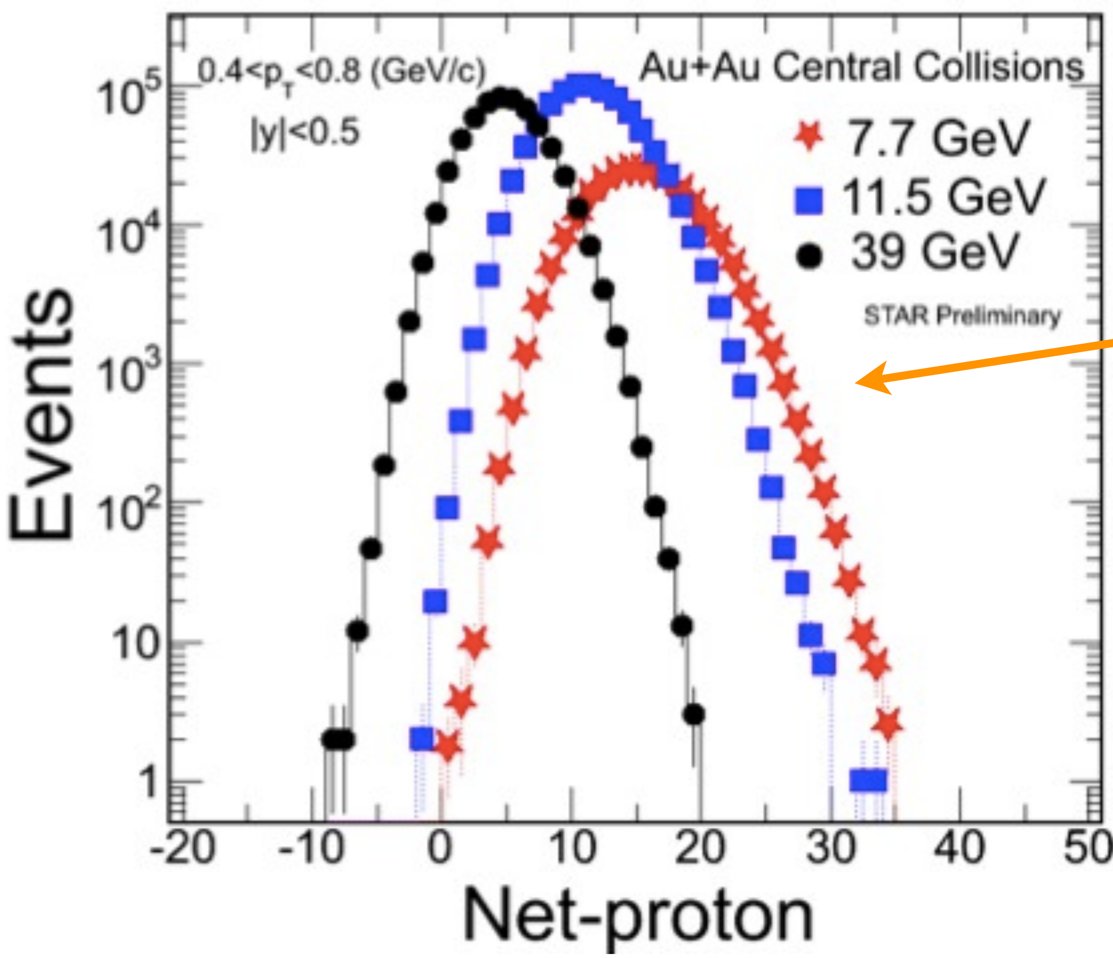
$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

where $$Z_n = \langle n | e^{-\beta H} | n \rangle$$

$$\xi = e^{\mu / T}$$ (Fugacity)

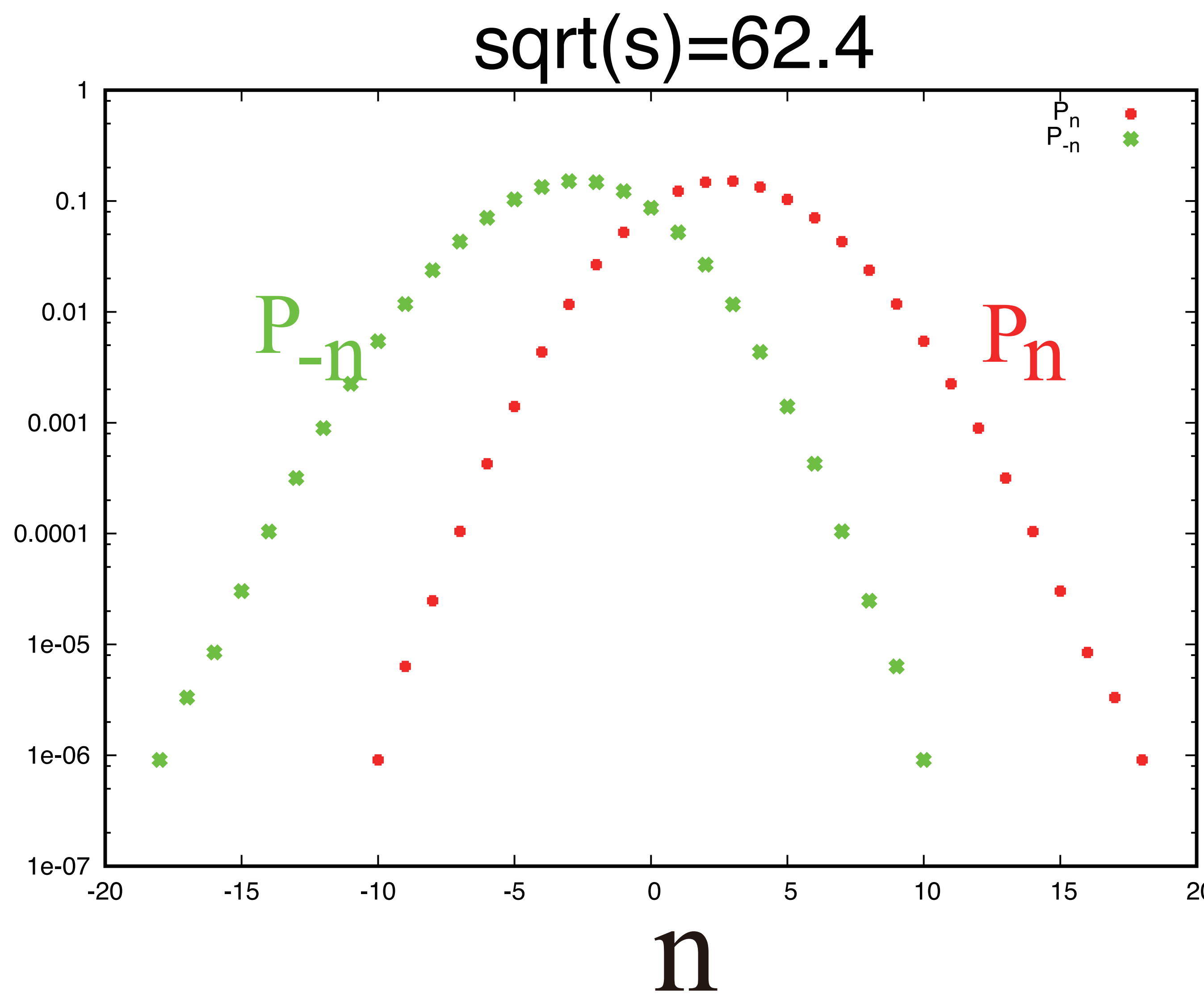
$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

Partition Function is
Sum of the Probability ...
If I know ξ, then I have Zn.



How can I obtain Zn from $P_n = Z_n \xi^n$?

ξ unknown ?!



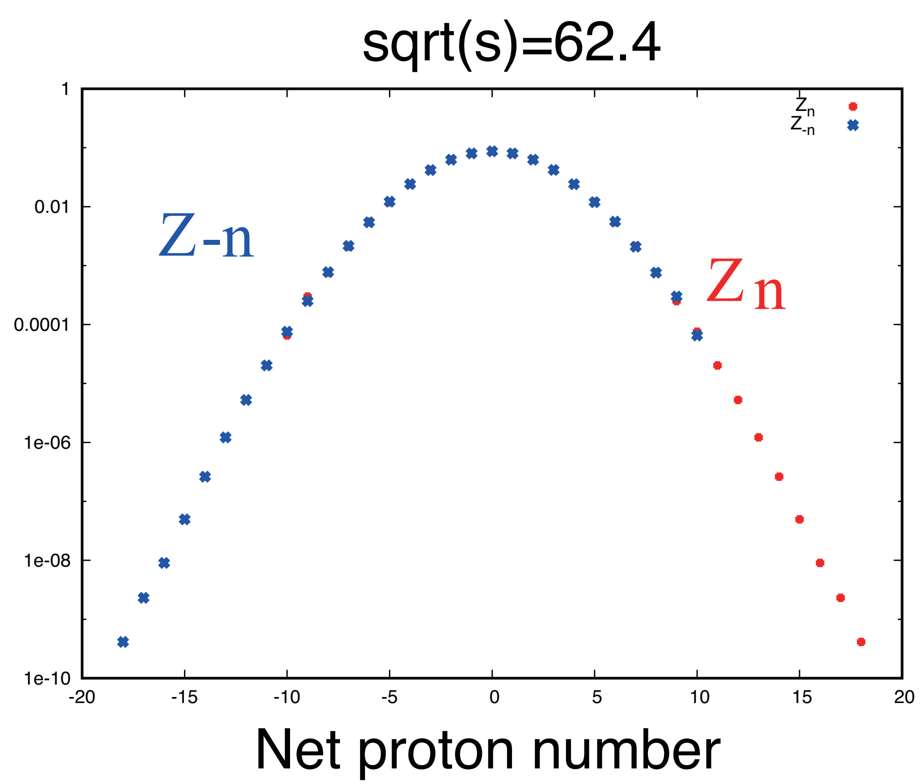
$$Z_n = P_n / \xi^n$$

Particle-AntiParticle Symmetry !

$$Z_{+n} = Z_{-n}$$

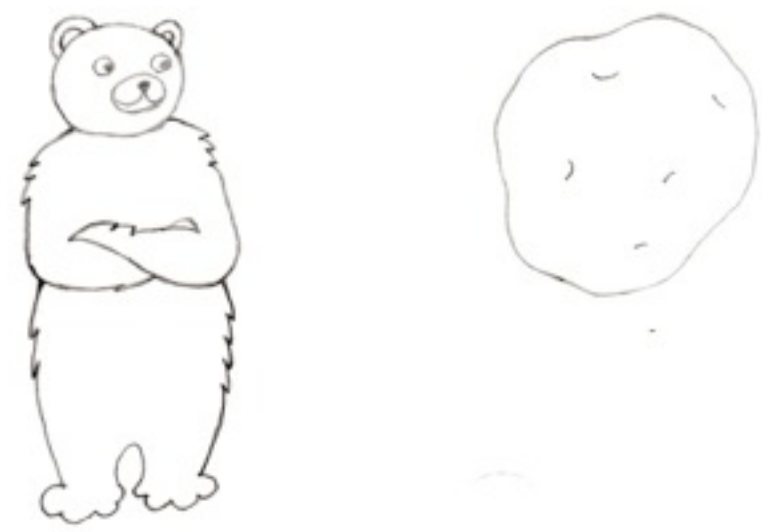
$$\xi = e^{\mu/T}$$

One Parameter !

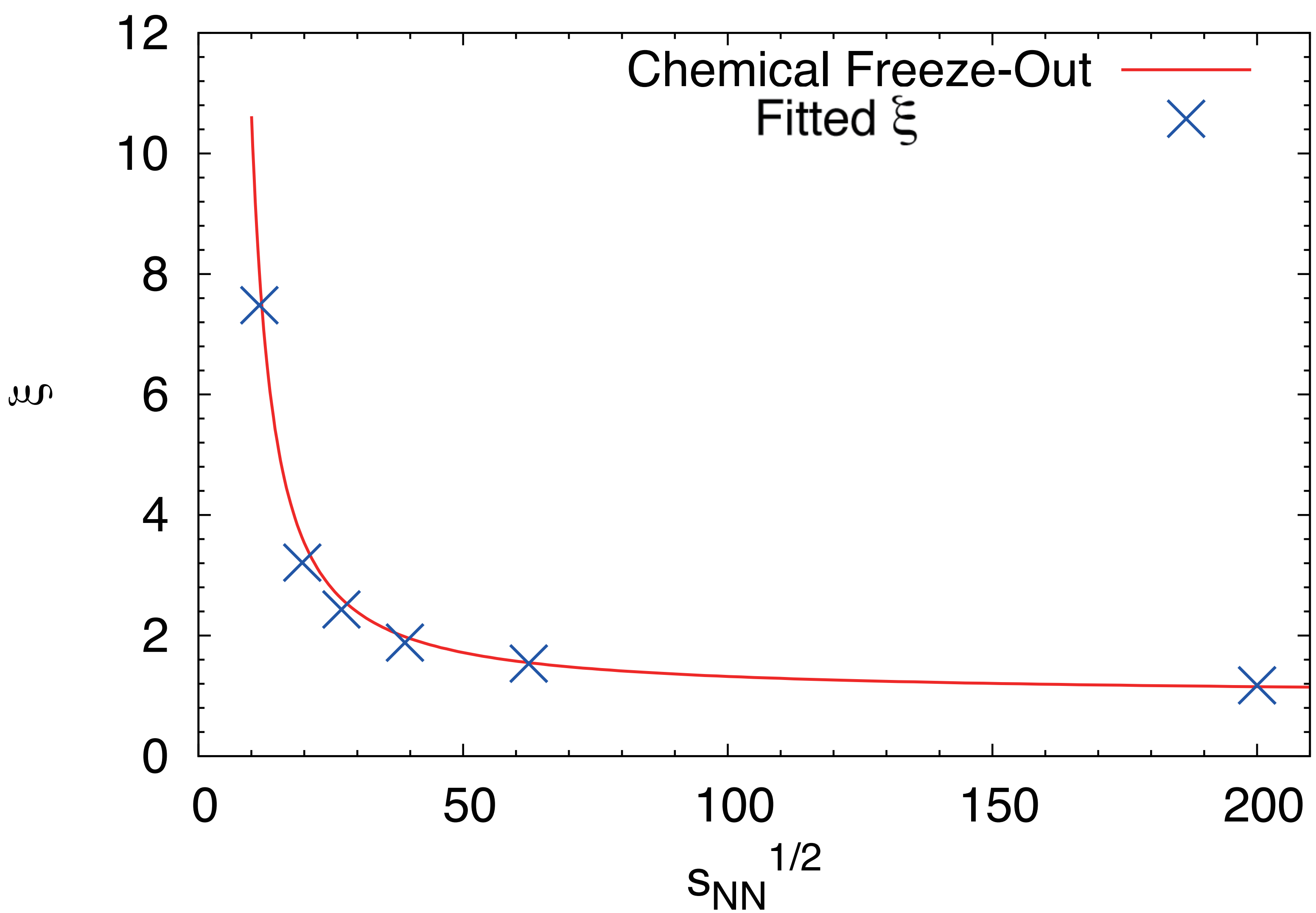


Fit ξ

Potato -UntiPotato Symmetry ?



Fitted ξ are very consistent with those by Freeze-out Analysis.



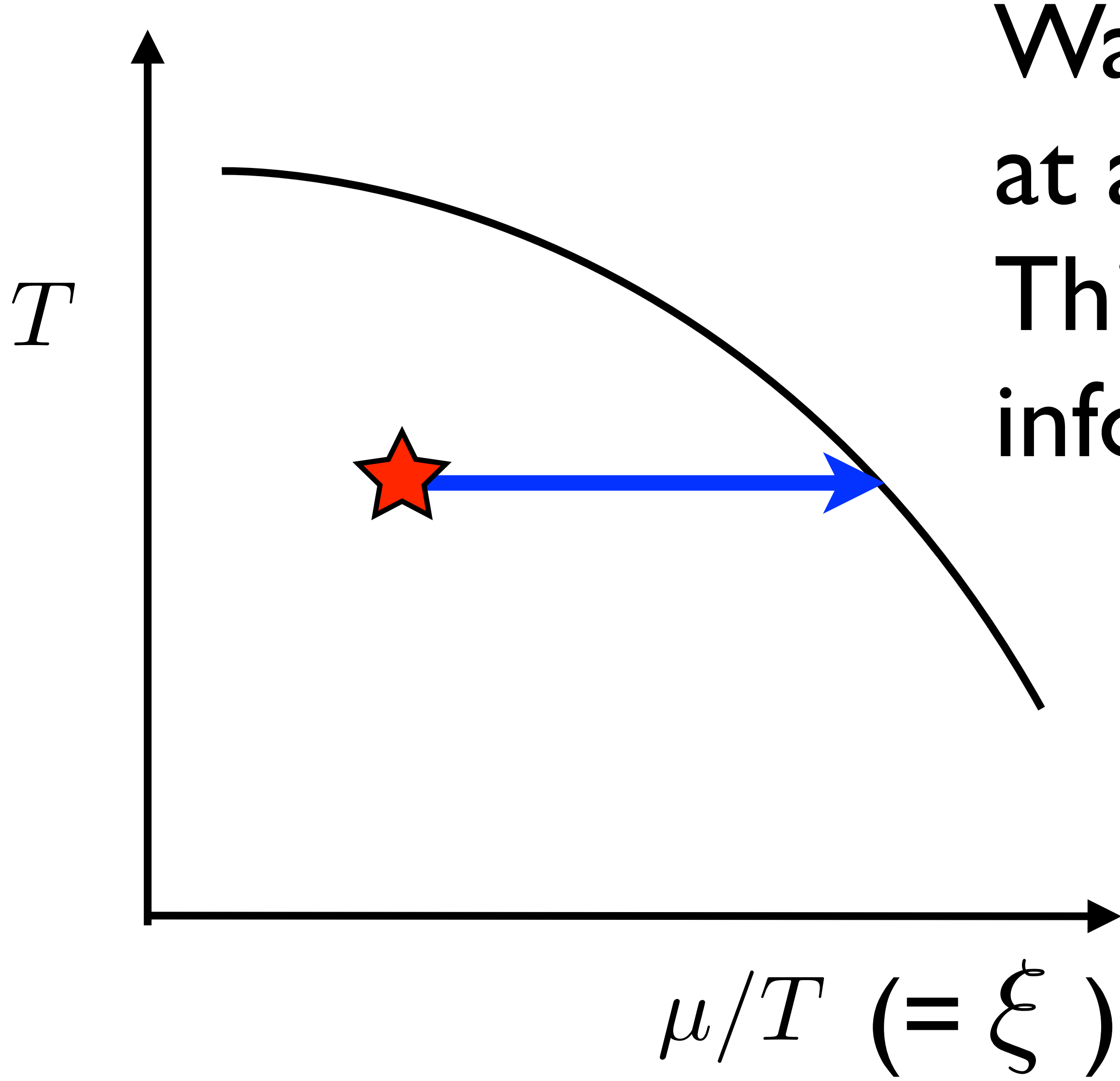
x This work

— Freeze-out

J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

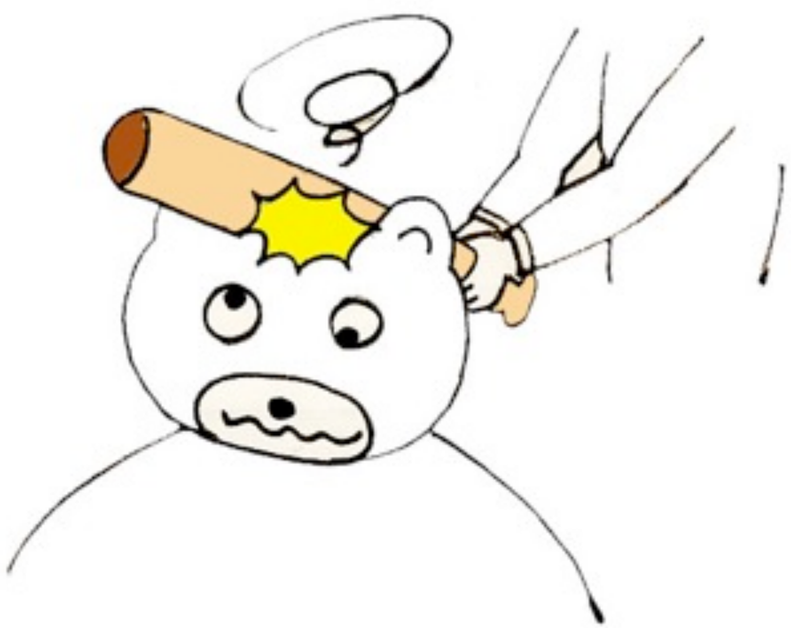
Now we have Zn of RHIC data
(sqrt(s)=9.6, 27, 39, 62.4, 200 GeV)



Wao ! We can calculate
at any density !
This includes all QCD Phase
information !

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Do not forget that your *n* is finite !

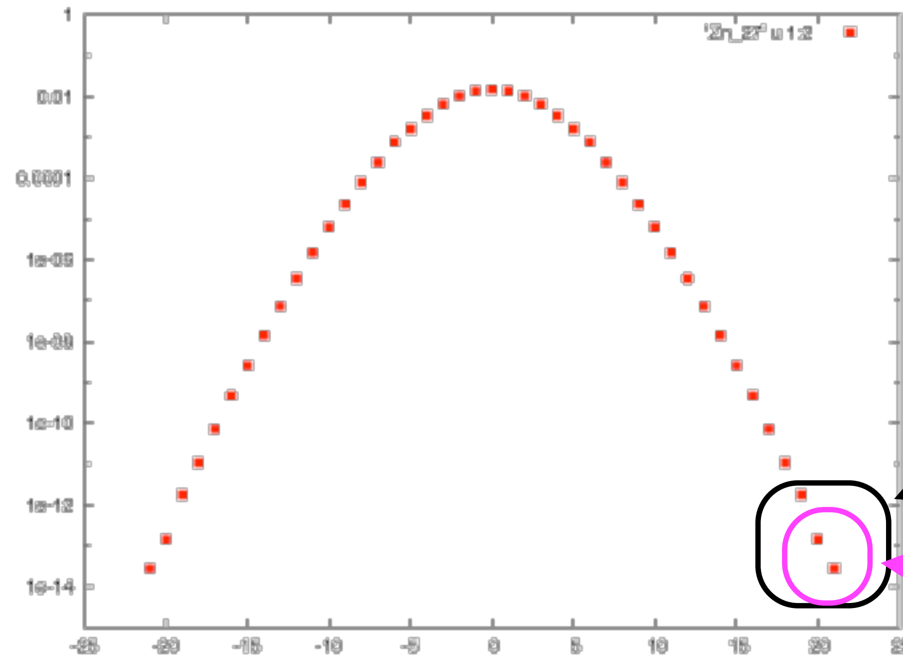


Hunting the QCD Phase Transition Regions

Find Rooms where No Criminal.

⇒ The Target is in other Room.

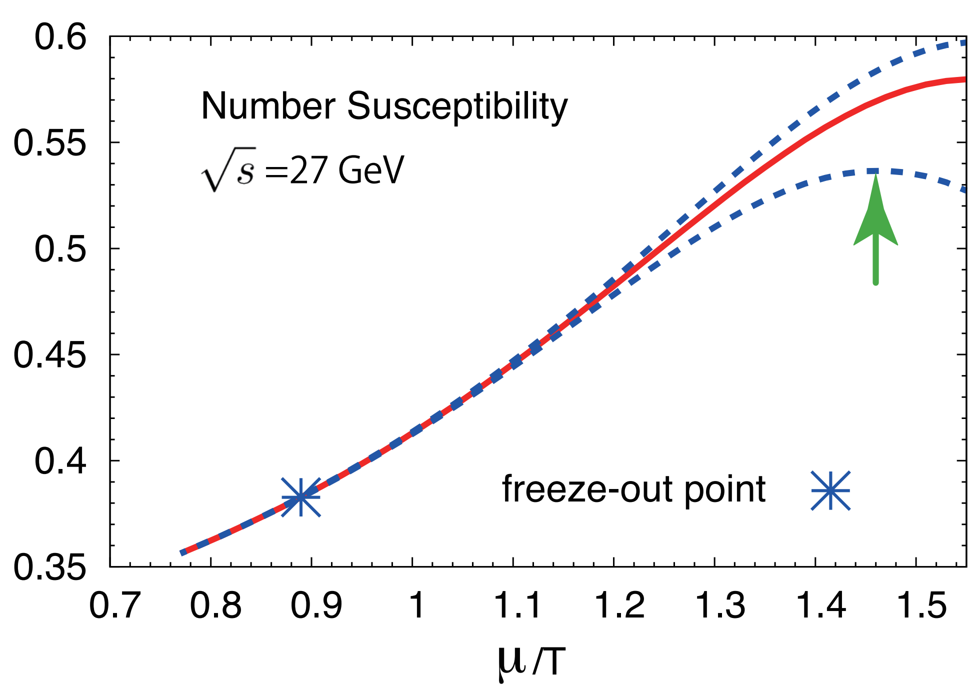
Not here ! Then, ..

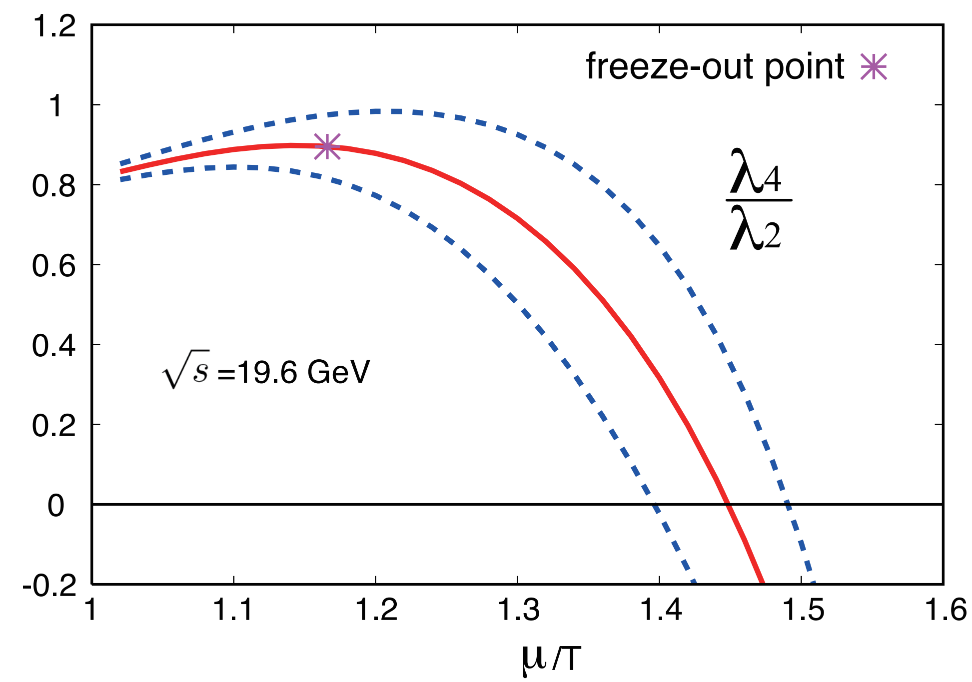


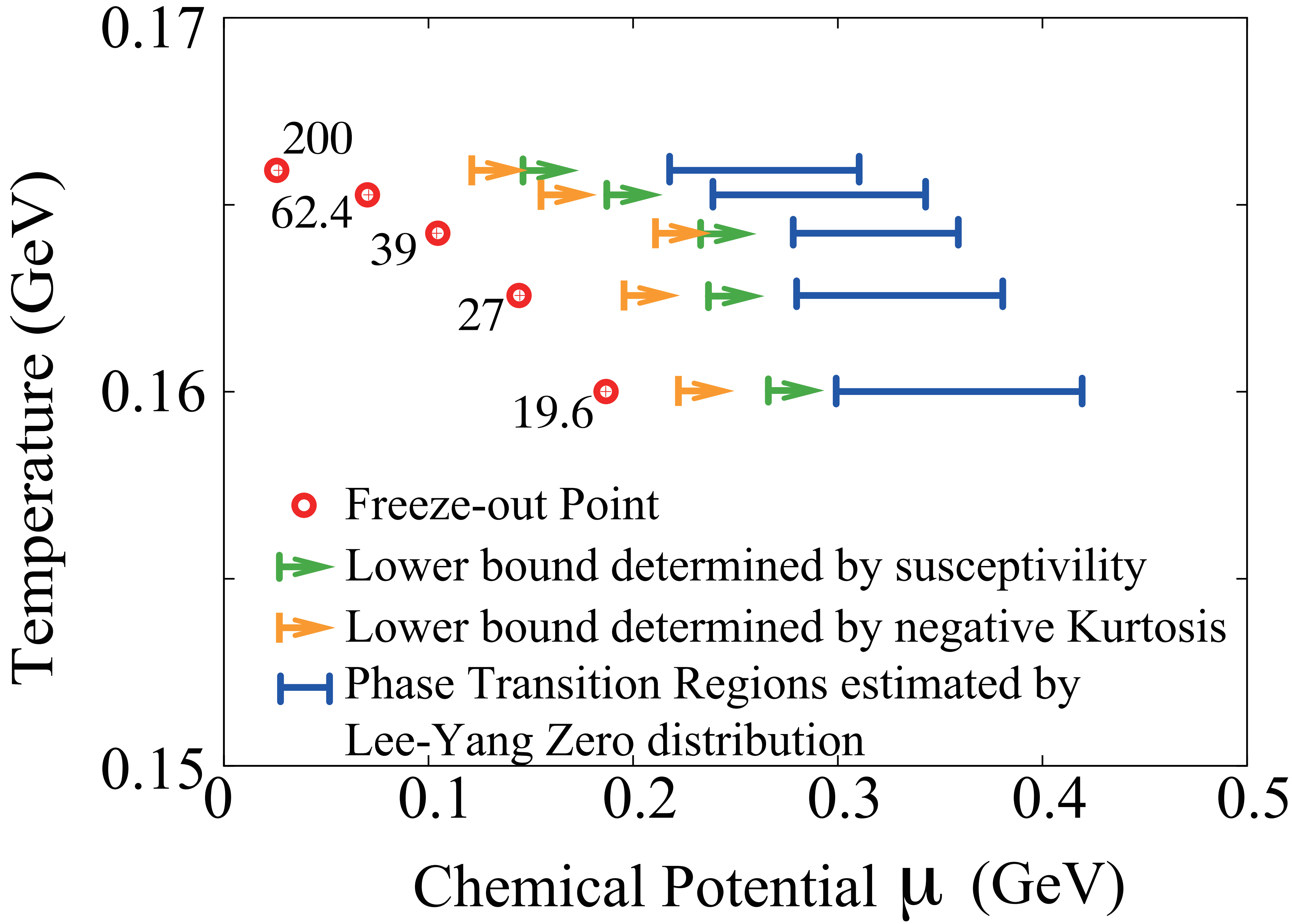
What happens

if we increase these points 15%

if we drop these points







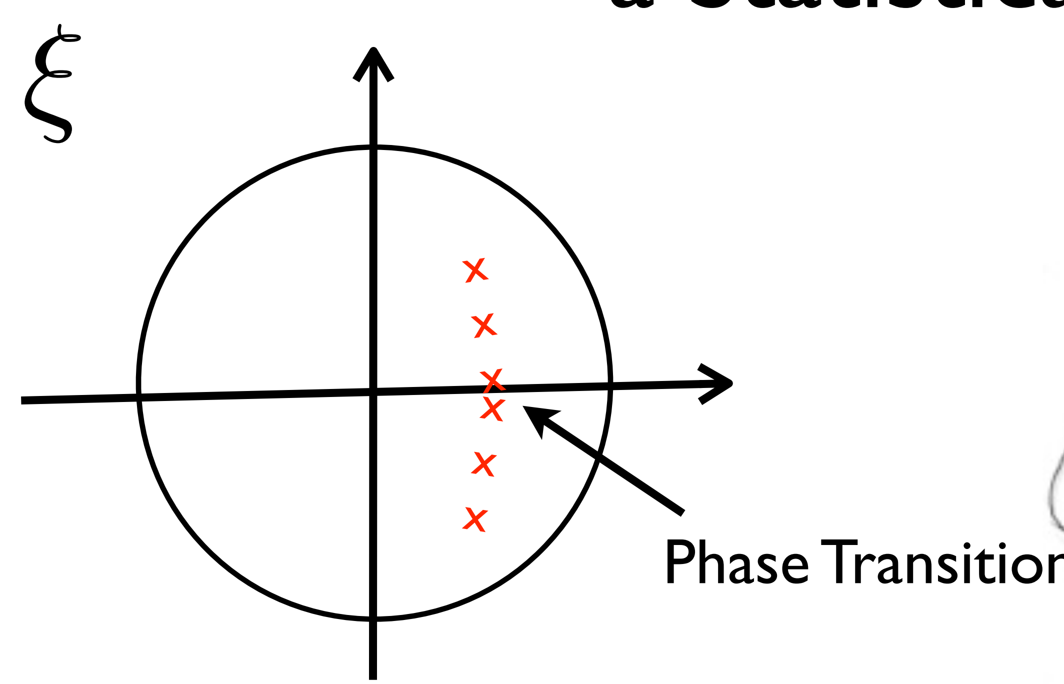
0.17
0.16
0.15
200
62.4
39
27
19.6
Freeze-out Point
Lower bound determined by susceptivility
Lower bound determined by negative Kurtosis
Phase Transition Regions estimated by
Lee-Yang Zero distribution
0
0.1
0.2
0.3
0.4
0.5
Temperature (GeV)
Chemical Potential μ (GeV)

Lee-Yang Zeros

Zeros of Zn in Complex Fugacity Plance.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System



Simple Factorization Problem ?

$\xi^{N_{max}/2}$ $\xi^{N_{max}/2}$

$$Z(\xi,T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

$$= \prod_k (\xi - \alpha_k) = 0$$

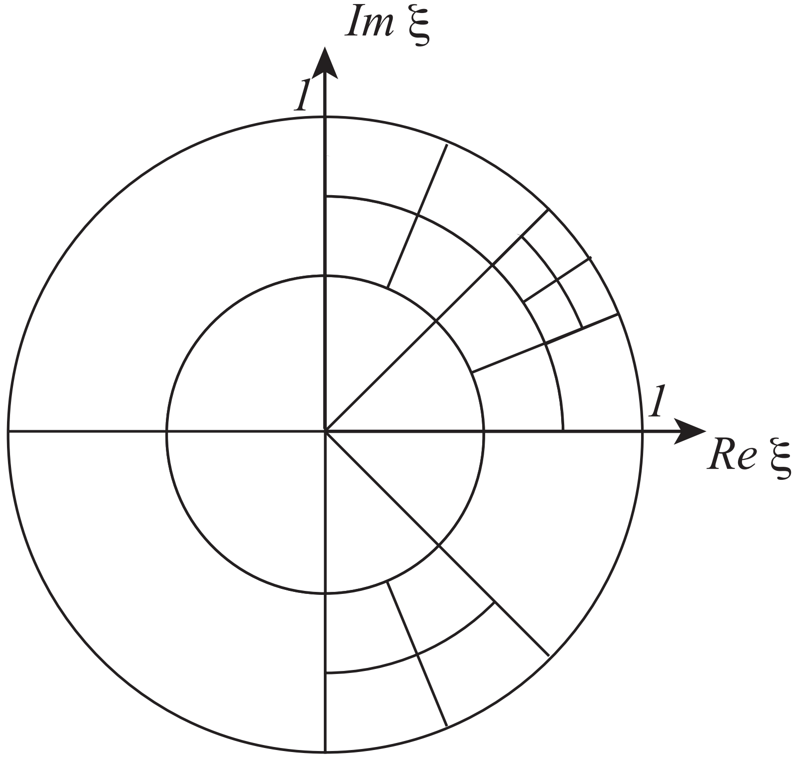
For large Nmax, this is an ill-posed problem.

IMSL Library does not work.

Minimum Search of $|Z(\xi)|$

Difficult to confirm a real Zero from very small value.

cut Baum-Kuchen (cBK) Algorithm



$$f(\xi) = \prod_k (\xi - \alpha_k)$$

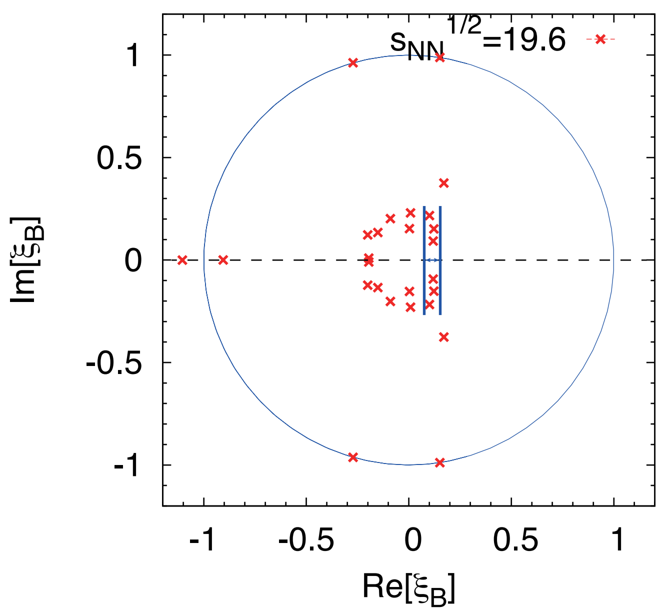
$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

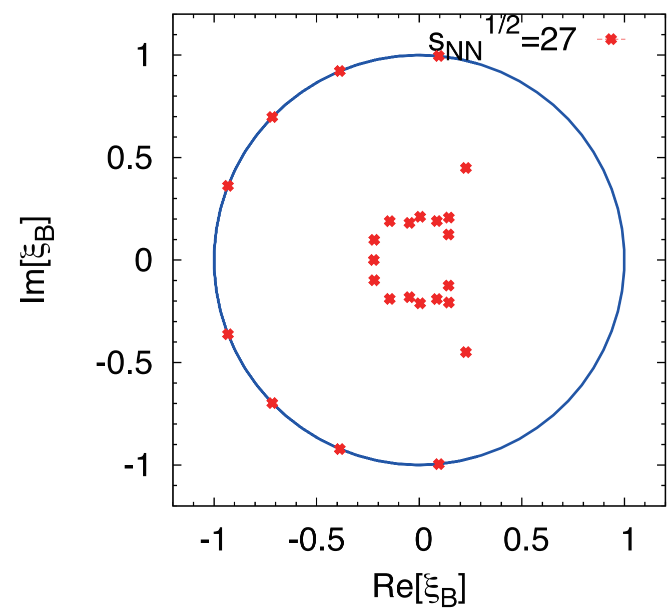
$$\frac{1}{2\pi} \oint_C \frac{f'}{f} d\xi =$$ Number of Zeros in Contour C

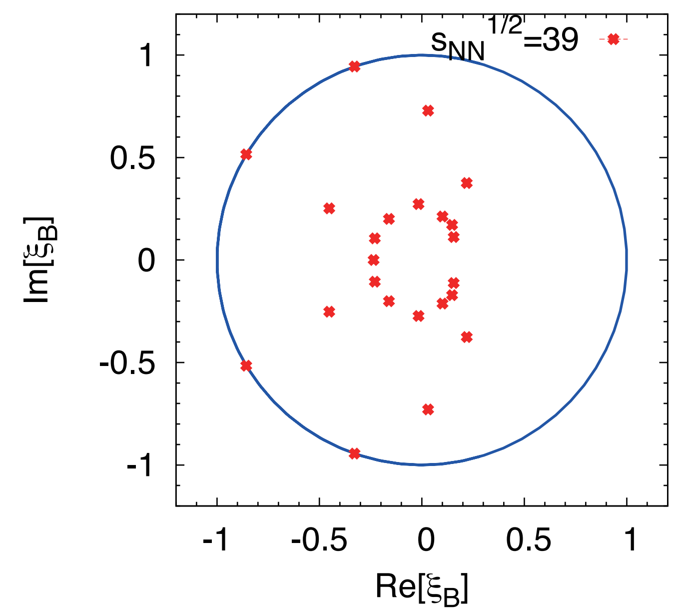
50 - 100 number of significant digits

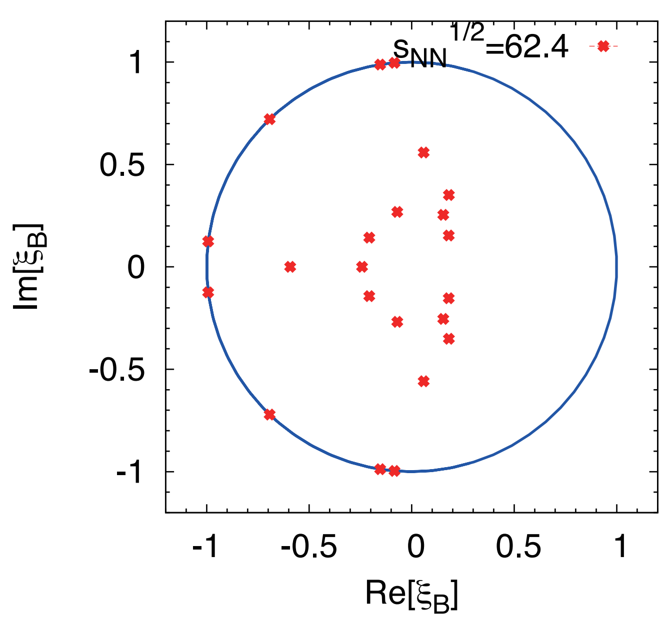
A Coutour is cut into four pieces if there are zeros insied.

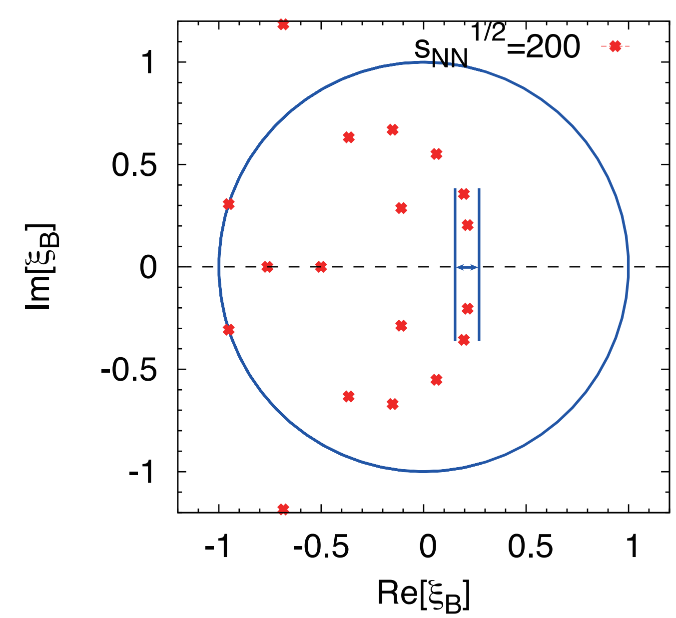
LYZ: RHIC Experiments











Lattice

$$Z_{GC}(\mu) = \int DU \left[\frac{C_0 \sum c_n \xi^n}{\det \Delta(0)}\right]^{N_f} (\det \Delta(0))^{N_f} e^{-S_G}$$

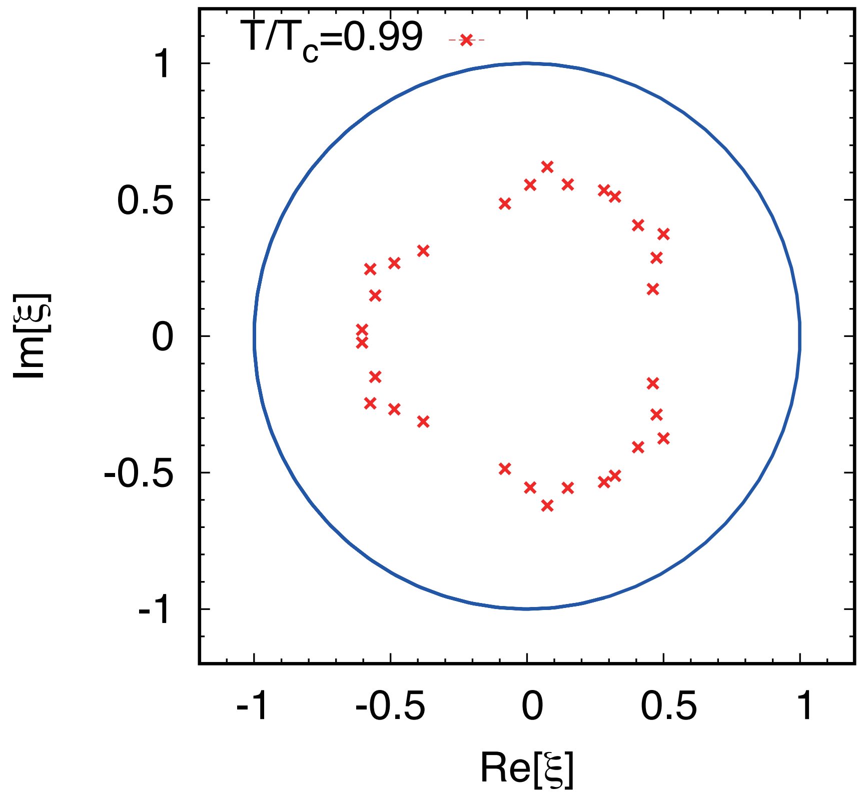
$$\int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

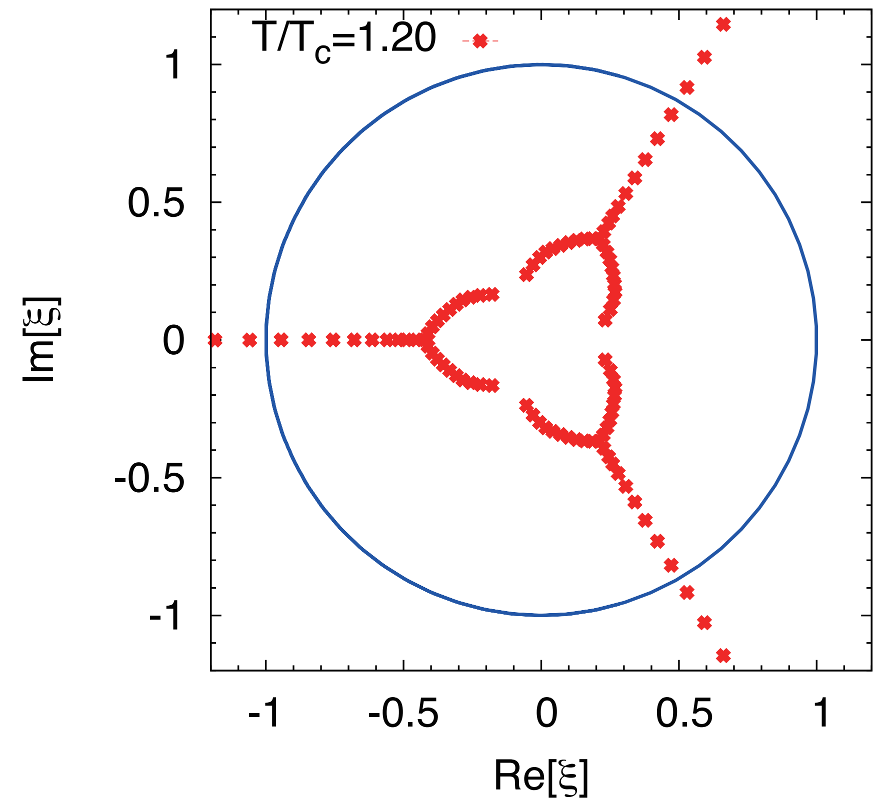
$$= \int DU (\sum a_n \xi^n)(\det \Delta(0))^{N_f} e^{-S_G}$$

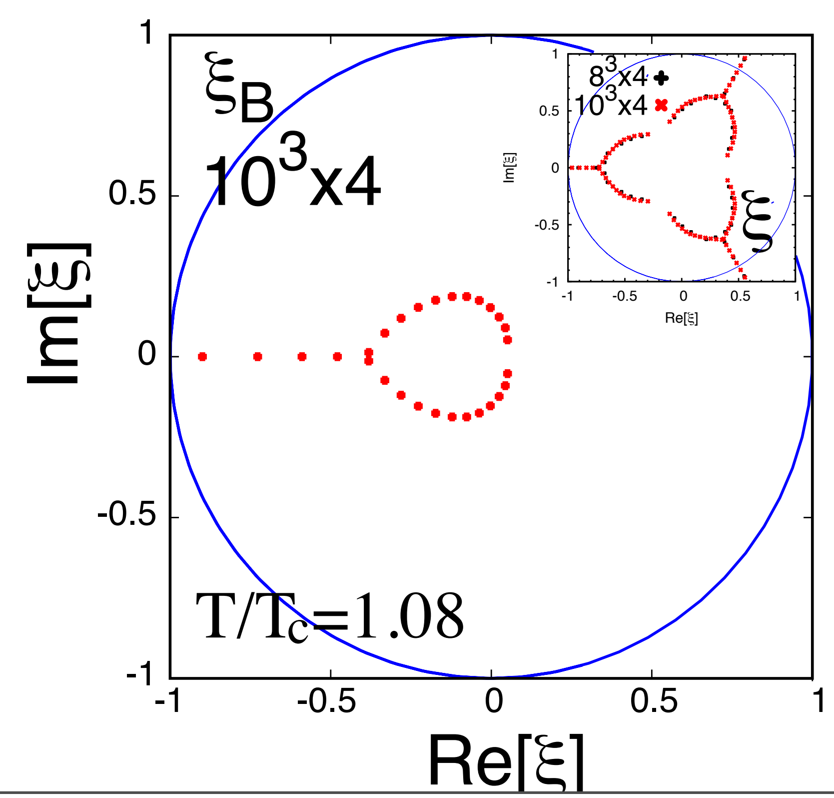
$$= \sum_n \xi^n \int DU a_n (\det \Delta(0))^{N_f} e^{-S_G}$$

$$Z_{GC}(\mu) = \sum Z_n \xi^n$$

LYZ: Lattice QCD







Lattice: How to Calculate

$$Z_(\mu, T) = \int \mathcal{D}U \;\; (\det\Delta(\mu))^{\mathrm{N_f}} \exp(-S_G)$$

$$\det\Delta(\mu) \to \left(\frac{\det\Delta(\mu)}{\det\Delta(0)}\right) \det\Delta(0)$$

$$\det\Delta(\mu) \propto \xi^{-N_{\mathrm{red}}/2} \prod_{n=1}^{N_{\mathrm{red}}} (\lambda_n + \xi)$$

$$\propto \sum_{n=-N_{\mathrm{red}}/2}^{N_{\mathrm{red}/2}} c_n \xi^n$$

Fugacity Expansion
Nagata and A. Nakamura,
Phys. Rev. D82 094027

Skellam

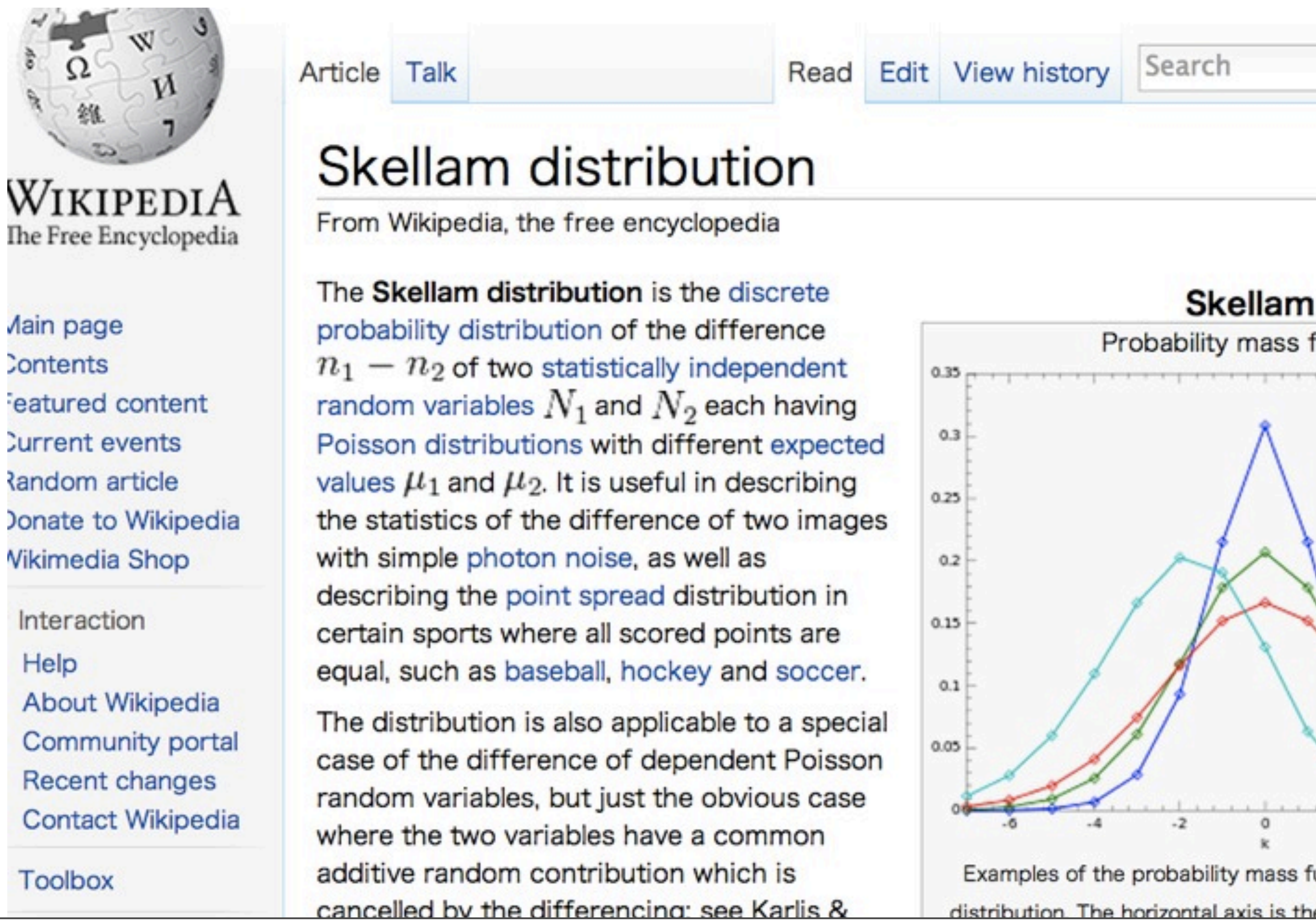
Skellamの場合
はどうなるん
ですか？

大変だ、新しい話だ
勉強しないと！

Skellam

新しくないよ

Skellam, J. G. (1946)
"The frequency distribution of the difference between two Poisson variates belonging to different populations". *Journal of the Royal Statistical Society, Series A*, 109 (3), 296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Skellam distribution

From Wikipedia, the free encyclopedia

The **Skellam distribution** is the discrete probability distribution of the difference $n_1 - n_2$ of two statistically independent random variables N_1 and N_2 each having Poisson distributions with different expected values μ_1 and μ_2. It is useful in describing the statistics of the difference of two images with simple photon noise, as well as describing the point spread distribution in certain sports where all scored points are equal, such as baseball, hockey and soccer.

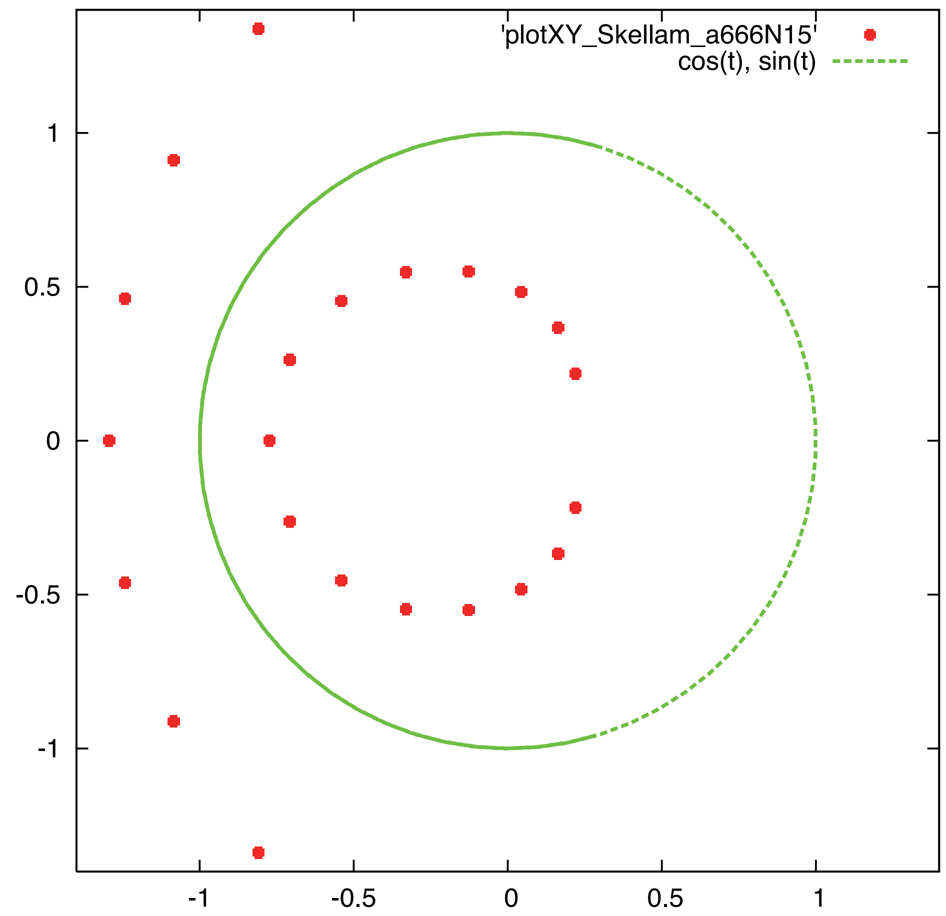
The distribution is also applicable to a special case of the difference of dependent Poisson random variables, but just the obvious case where the two variables have a common additive random contribution which is cancelled by the differencing: see Karlis &

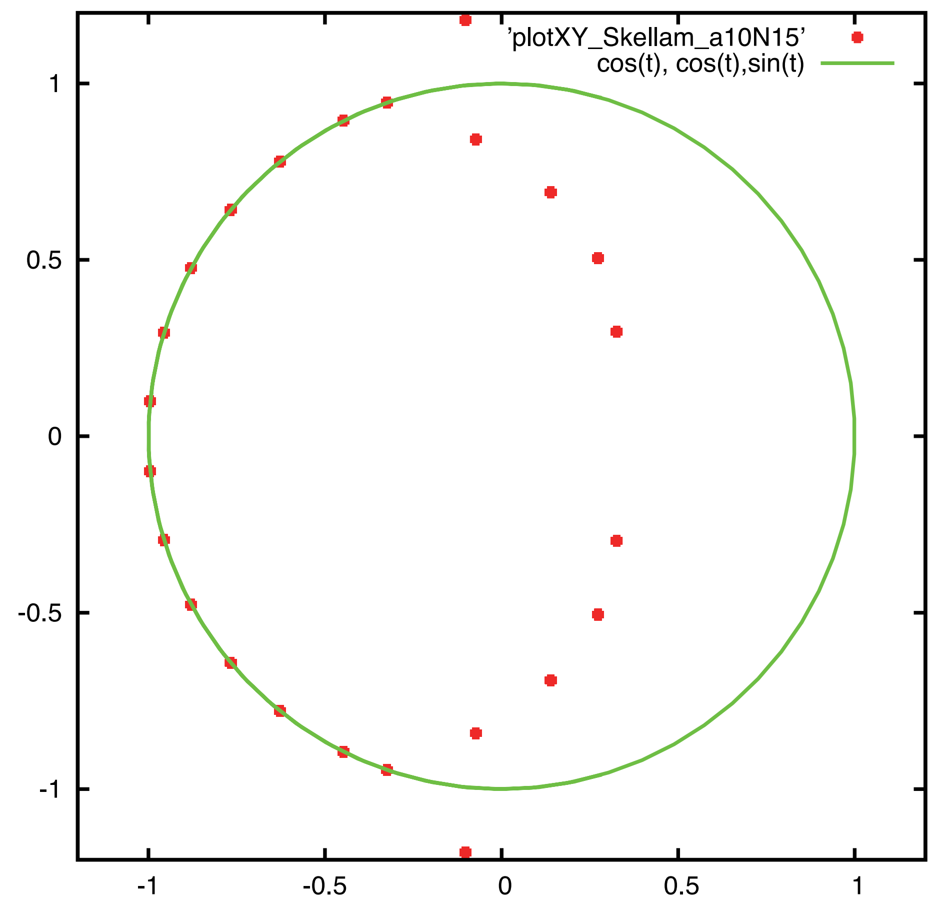
Skellam

Probability mass fu

Examples of the probability mass fu
distribution. The horizontal axis is the

LYZ : Skellam


'plotXY_Skellam_a666N15'
cos(t), sin(t)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


'plotXY_Skellam_a10N15'
cos(t), cos(t),sin(t)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Conclusion

- メチャクチャ楽しい仕事だった
- 突っ込みどころはまだたくさん
  - proton multiplicityは保存量じゃないだろう
  - Lee-Yang zeros分布とO(4)スケーリングの関係は？
  - LatticeはT<Tcがちゃんと計算できてない
- 実験でNmaxを大きくしてくれれば、QCD相図はかなり押さえ込まれる
- 格子でTc以下を計算しなければ
  - 虚数化学ポテンシャル領域の計算だが大きなノイズ