極限物質の性質を決めるには?

超新星：状態方程式データテーブル

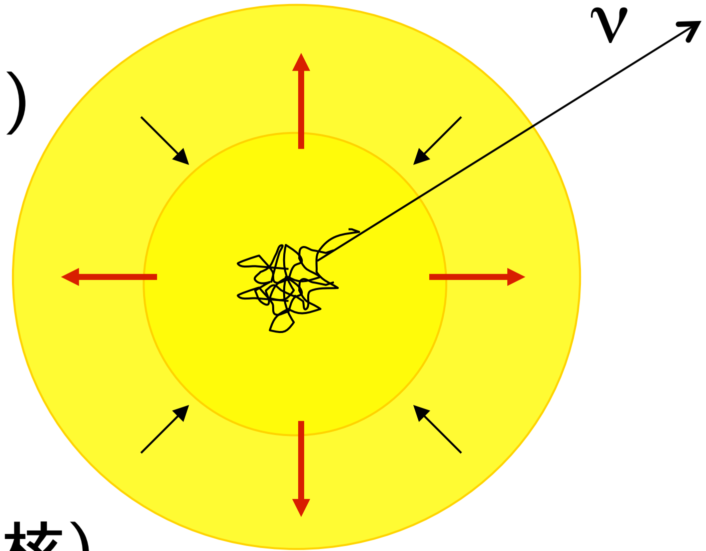
中性子星と超新星の状態方程式

- 中性子星
  - 密度のみの関数
  - ほぼ中性子物質
  - ゼロ温度
  - 冷えた中性子星
- 多くの状態方程式
  - 原子核実験
  - 中性子星質量・半径
- 超新星
  - 密度だけでなく
  - 電子の割合が変わる
  - 有限温度
  - 超新星爆発時
- 少ないデータテーブル
  - 数値シミュレーション
  - 中性子星合体にも

極限状態での物質の性質

状態方程式(Equation of State; EOS)

1. 圧力-密度
   - 星の構造、ダイナミクス
2. 温度(エントロピー)
   - ν-エネルギー分布, 平均エネルギー
3. 組成(陽子, 中性子, ヘリウム, 原子核)
   - 原子核反応、ニュートリノ反応率



- 状態方程式データテーブル
  - 密度: $10^0 \sim 10^{15}$ g/cm^3
  - 陽子の割合: $0 \sim 0.6$
  - 温度: $0 \sim 100$ MeV
- $(\rho, T, Y_e) : \varepsilon, p, S, \mu_n, \mu_p, X_i, m^*, \ldots$

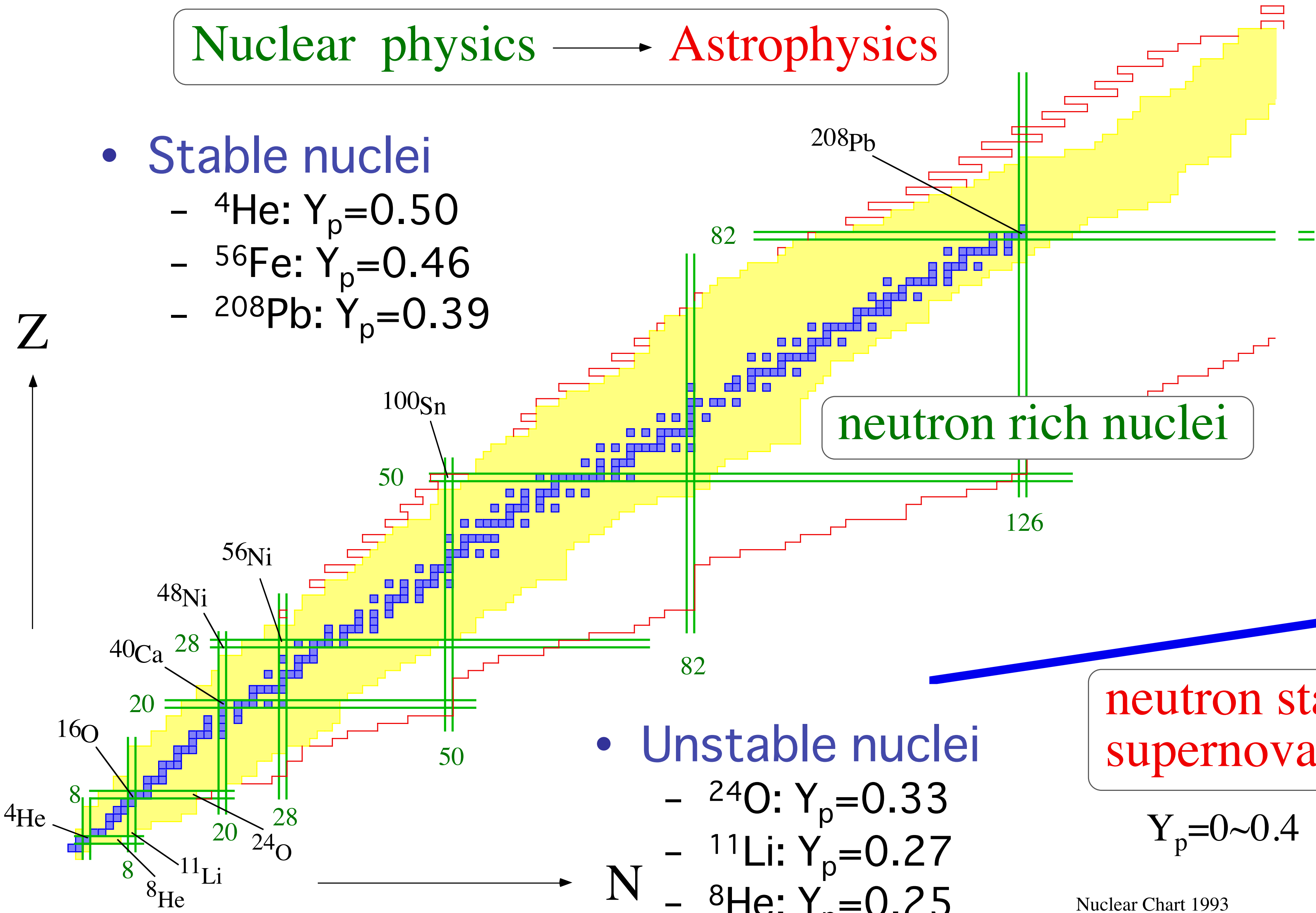
ちなみに

原子核物質密度:
$n_0 = 0.17$ fm^{-3}
($\rho_0 = 3 \times 10^{14}$ g/cm^3)

温度:
1 MeV ~ 10^{10} K

Nuclear physics ⟶ Astrophysics

- Stable nuclei
  - ^{4}He: Y_p=0.50
  - ^{56}Fe: Y_p=0.46
  - ^{208}Pb: Y_p=0.39



- Unstable nuclei
  - ^{24}O: Y_p=0.33
  - ^{11}Li: Y_p=0.27
  - ^{8}He: Y_p=0.25

neutron stars
supernovae

Y_p=0~0.4

Nuclear Chart 1993
by Chiharu Tanihata

65% for one page

理化学研究所
RIビームファクトリー

不安定原子核を作り出す加速器施設

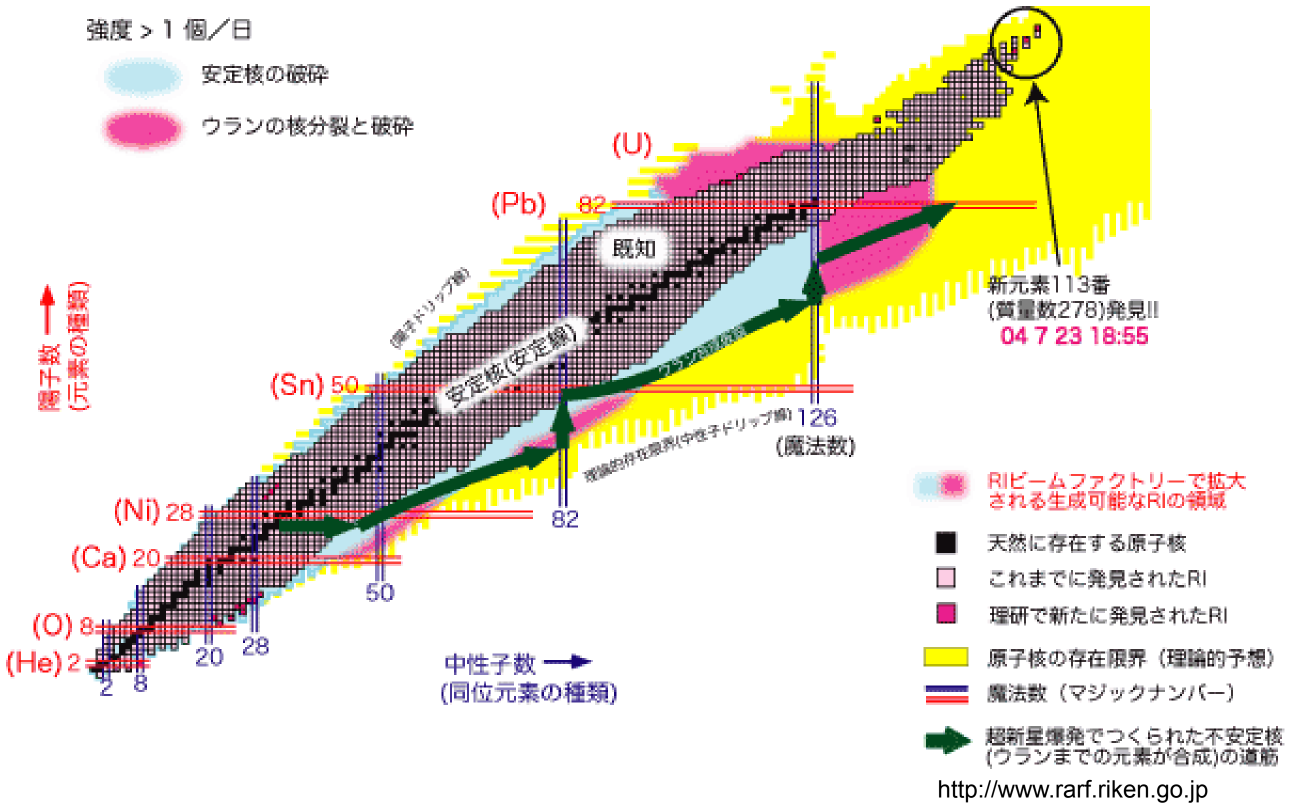
自然界には存在しない原子核

宇宙/星の進化に登場する原子核

極限状態での原子核の研究が可能

^{131}Agの発見(2013)
Z=47, N=84
Z/A=0.358

超新星コアには近い



超新星爆発における状態方程式

- 系統的な研究のための状態方程式
  - Takahara-Sato
  - Baron-Cooperstein-Kahana
- 数値シミュレーション用の状態方程式データ
  - Wolff-Hillebrandt
  - Lattimer-Swesty EOS (LS)
    - *質量公式の拡張*
  - Relativistic EOS (Shen)
    - *相対論的核子多体理論(RMF), 不安定核データ*
- 状態方程式データの改良
  - GShen, Hempel, Furusawa
    - 3次元計算、多核種混合、相互作用改善など
  - Non-relativistic EOS
    - *変分法による核子多体理論, 核子間ポテンシャル*
- ハドロン・クォーク物質への拡張
  - Hyperon EOS
    - RMF+Λ, Σ, Ξ粒子の混入
  - Quark-Hadron phase transition
    - RMF+Bag model

- 広い範囲の密度・温度・組成
- 一貫した枠組みで取り扱う
- 実験データによるチェック

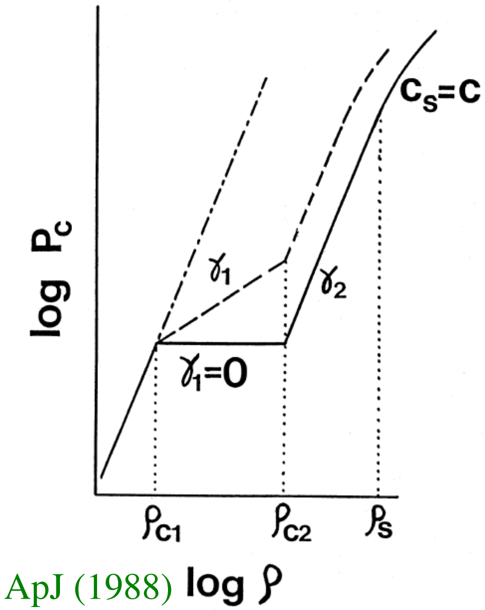
状態方程式の違い
→ ダイナミクス・ν反応
→ 爆発メカニズムへの影響

- Baron-Cooperstein-Kahana EOS
  - Analytic formula $$P = \frac{K\rho_0}{9\gamma}\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right]$$
  - Polytrope at high density: $$P \propto \rho^{\gamma}$$
  - Behavior at neutron-rich $$K(Y_p) = K_{sym}[1-\alpha(1-2Y_p)^2]$$ $$\rho_0(Y_p) = \rho_0^{sym}[1-\beta(1-2Y_p)^2]$$

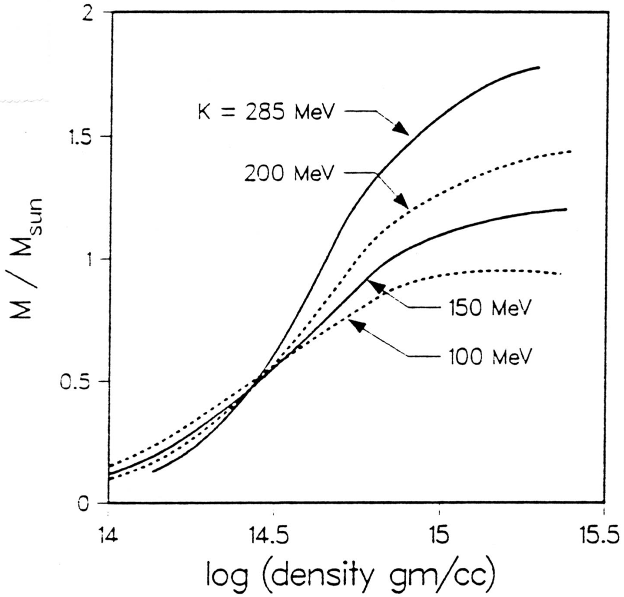
Incompressibility K
Adiabatic index

$$\Gamma = \frac{d\log P}{d\log \rho}$$

- Takahara-Sato EOS
  - Combination of polytrope $$P = C_i \rho^{\gamma_i}$$



Takahara Sato, ApJ (1988)



Glendenning, PRL (1986)

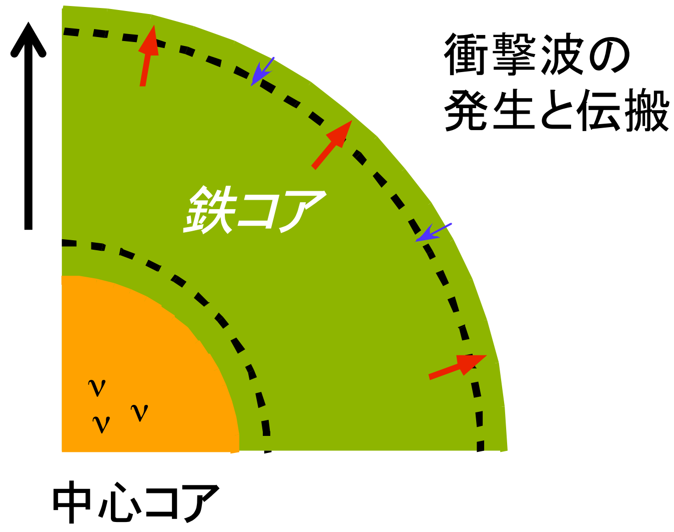
状態方程式の影響の例(1980年代後半)

- 解析的な式による状態方程式

Baron et al. PRL (1985)

$$p = \frac{K_0 \rho_0}{9\gamma}[(\rho/\rho_0)^\gamma - 1]$$

K_0: 非圧縮率, ρ_0: 核物質密度



- パラメータによる系統的な研究

Takahara (1985), Bruenn (1989)

- 状態方程式が柔らかい

K_0: 220 → 90 MeV

- 対称エネルギーが大きい

A_{sym}: 28 → 40 MeV

- 中心コアの質量が大きく、半径が小さい方が良い。
  →重力エネルギーの解放、鉄の分解での消費を抑える

•実際には、衝撃波は鉄のコアの途中で停滞(stall)してしまう。

Lattimer-Swesty equation of state

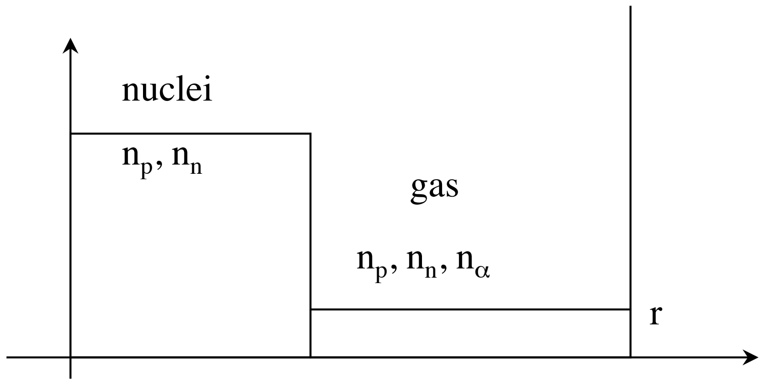
LS-EOS

Lattimer & Swesty Nucl. Phys. A535 (1991) 331

- Extension of compressible liquid drop model
  - **Energy of uniform matter is given by a function of density**

$$E_{bulk}(n,Y_p,T) = \Sigma \frac{\hbar^2 \tau_i}{2m} + [a + 4bx(1-x)]n^2 + cn^{1+\delta} - xn\Delta \qquad x = \frac{n_p}{n}$$

    - Based on Skyrme Hartree-Fock results
  - **Inputs: saturation properties**
    - B/A=16 MeV, A_{sym}=29.3 MeV, K=180, 220, 375 MeV
  - Two zone model (liquid+gas)
  - Minimize free energy
    - Under phase equilibrium
  - Subroutine



Relativistic equation of state for supernovae

Shen EOS *Shen, Toki, Oyamatsu & Sumiyoshi, 1998, NPA, PTP*

- Relativistic Mean Field + Local-Density Approx.
  - Based on relativistic Brueckner Hartree-Fock (RBHF) theory
  - Checked by exp. data of n-rich unstable nuclei
    - Nuclear structure: mass, charge radius, neutron skin,…
- EOS data table (~60MB) covers
  - Density: $10^5 \sim 10^{15.4}$ g/cm^3
  - Proton fraction: 0 ~ 0.56
  - Temperature: 0 ~ 100 MeV
- Extended studies on EOS table
  - With hyperons (Ishizuka-Tsubakihara-Ohnishi, 2006)
  - With quarks (Nakazato)
  - With mixed nuclei (Furusawa)

Relativistic Mean Field Theory - Effective Lagrangian

Serot, Walecka 1986

$$L_{RMF} = \overline{\Psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu} - e\gamma_\mu A^\mu\frac{1-\tau_3}{2}\right]\Psi$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4$$
$$-\frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \underline{\frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2}$$
$$-\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

← Rel. Brueckner HF

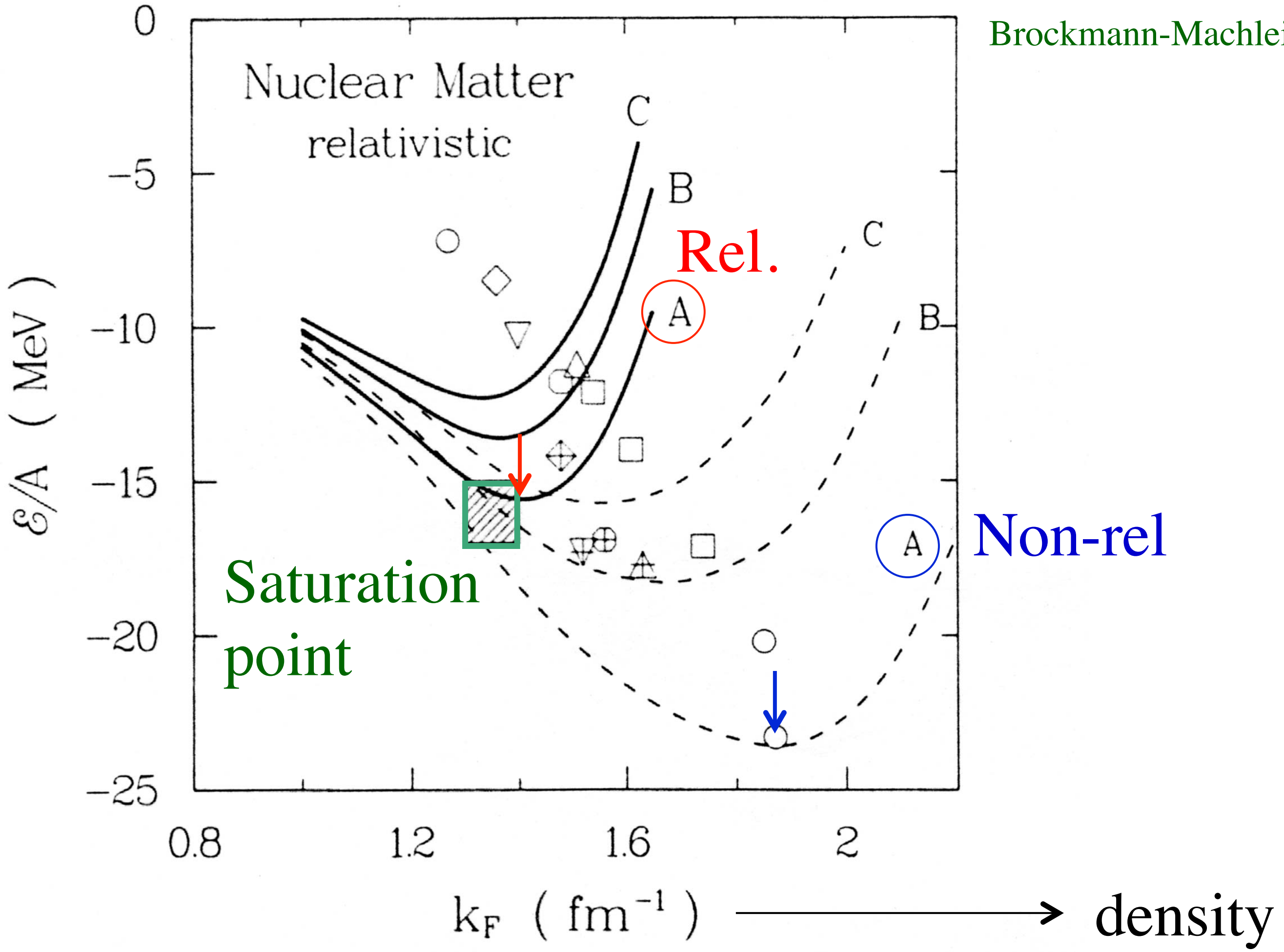
Parameters determined by nuclear data (masses, radii)

TM1: Sugahara, Toki Nucl. Phys. A 579 (1994) 557

Nuclear structure calculations ⟶ EOS calculations

Relativistic Brueckner Hartree-Fock Theory

Brockmann-Machleidt (1990)

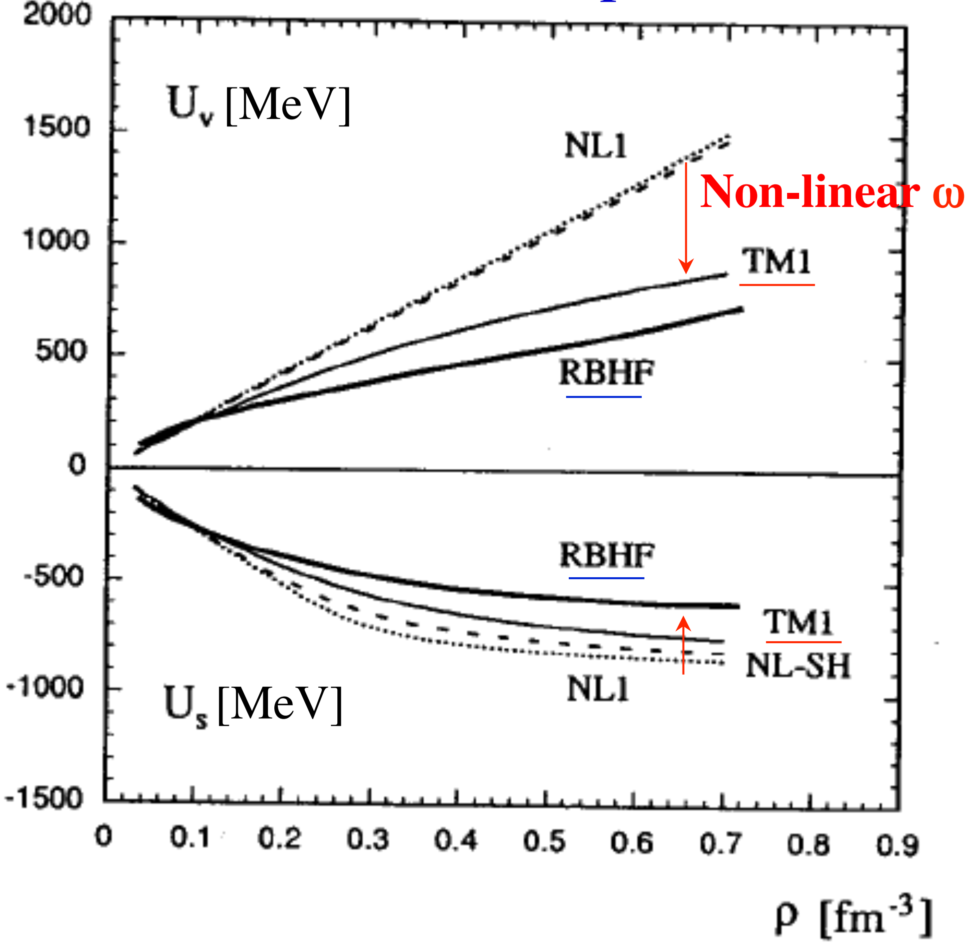


RMF lagrangian (σ, ω, ρ) with non-linear σ & ω terms

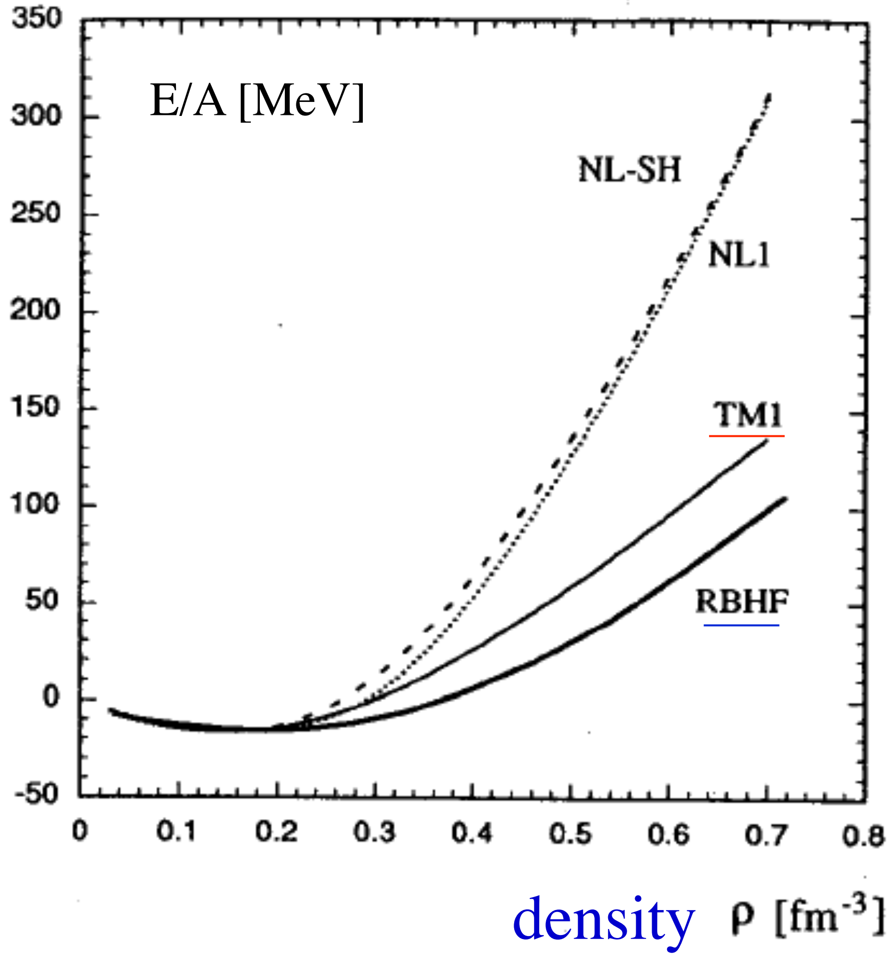
Serot-Walecka, Boguda-Bodmer, Sugahara

Non-linear ω term to reproduce behavior of rel. Brueckner HF (RBHF)

Scalar & vector potentials



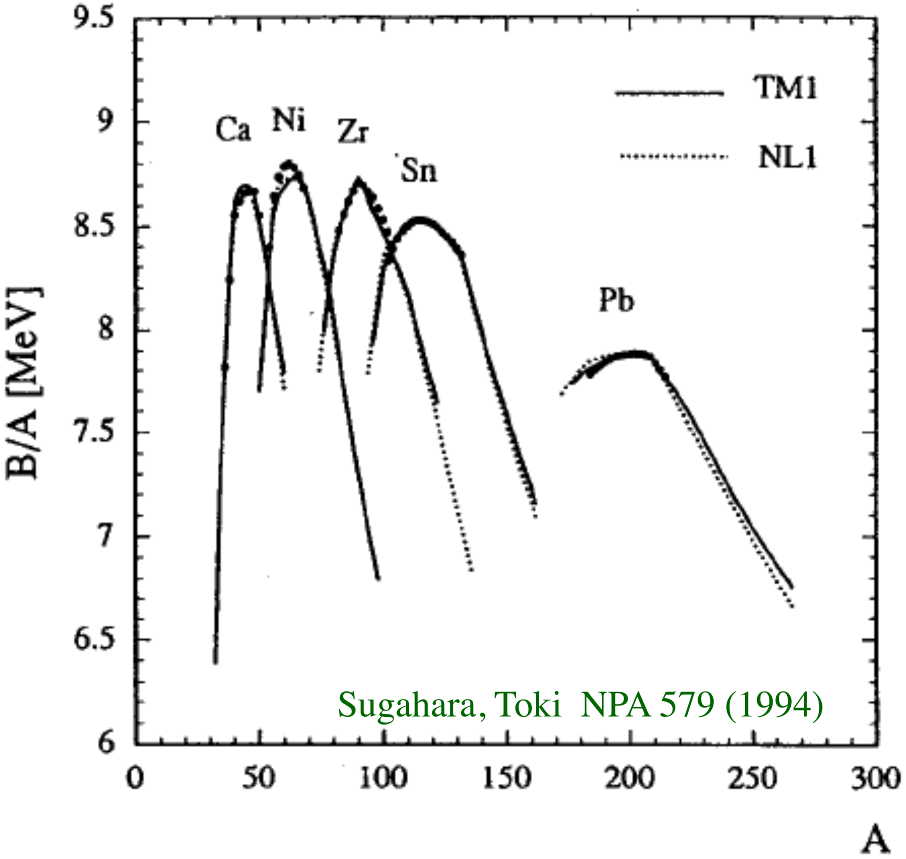
Nuclear matter EOS



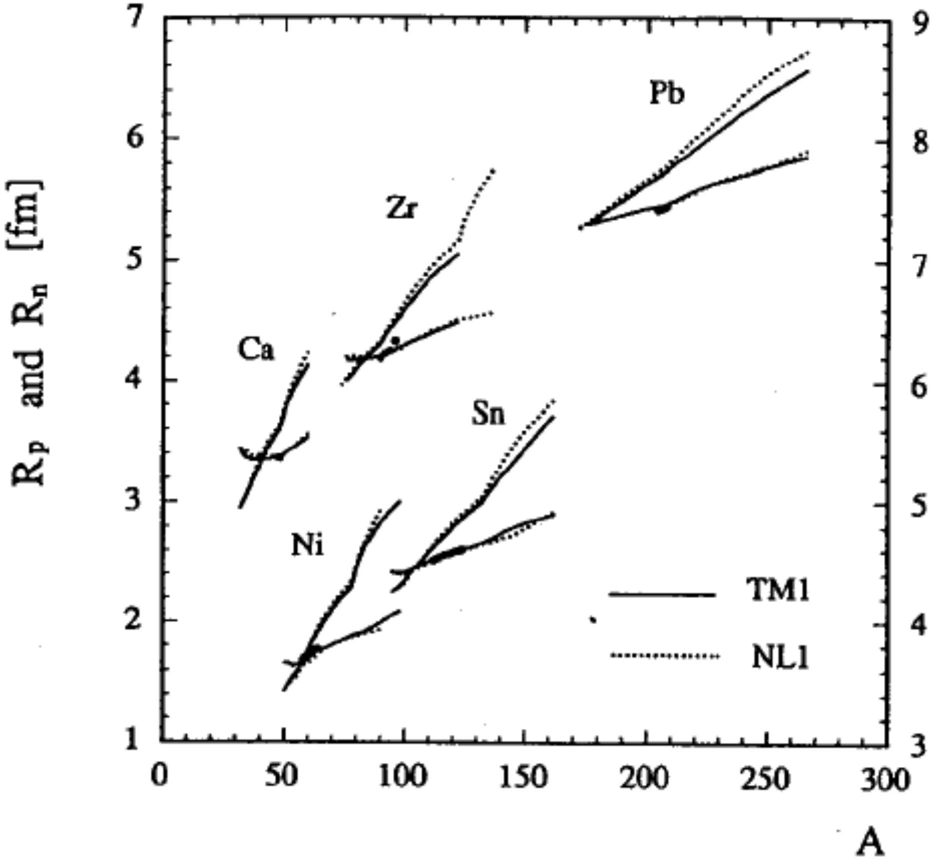
Sugahara, Toki Nucl. Phys. A 579 (1994) 557

Interaction determined by masses and radii: TM1

Binding energy of isotopes

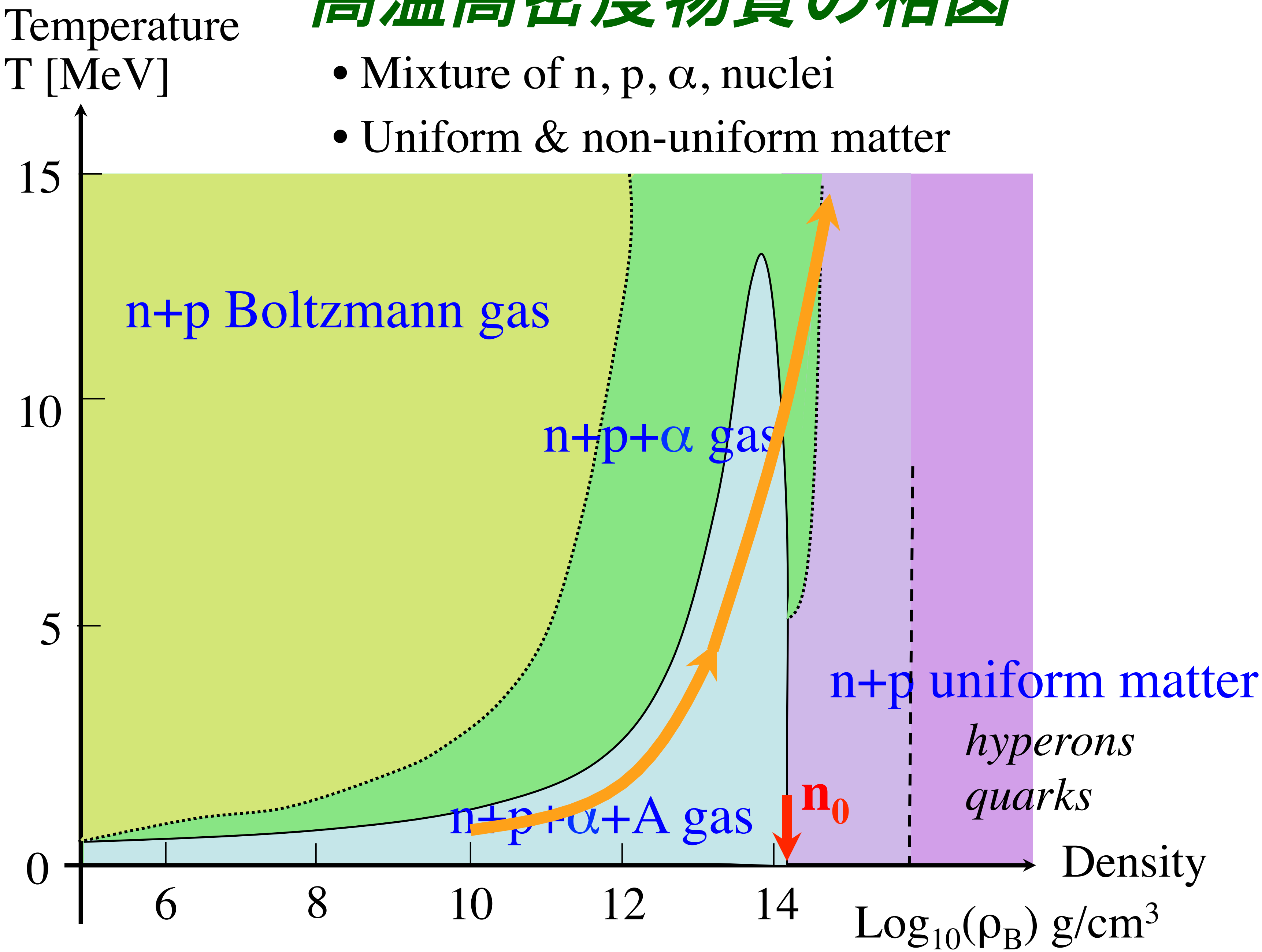


Matter radius of isotopes



高温高密度物質の相図

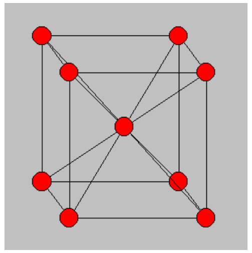
- Mixture of n, p, α, nuclei
- Uniform & non-uniform matter



Local density approximation in cell

Wigner-Seitz Cell

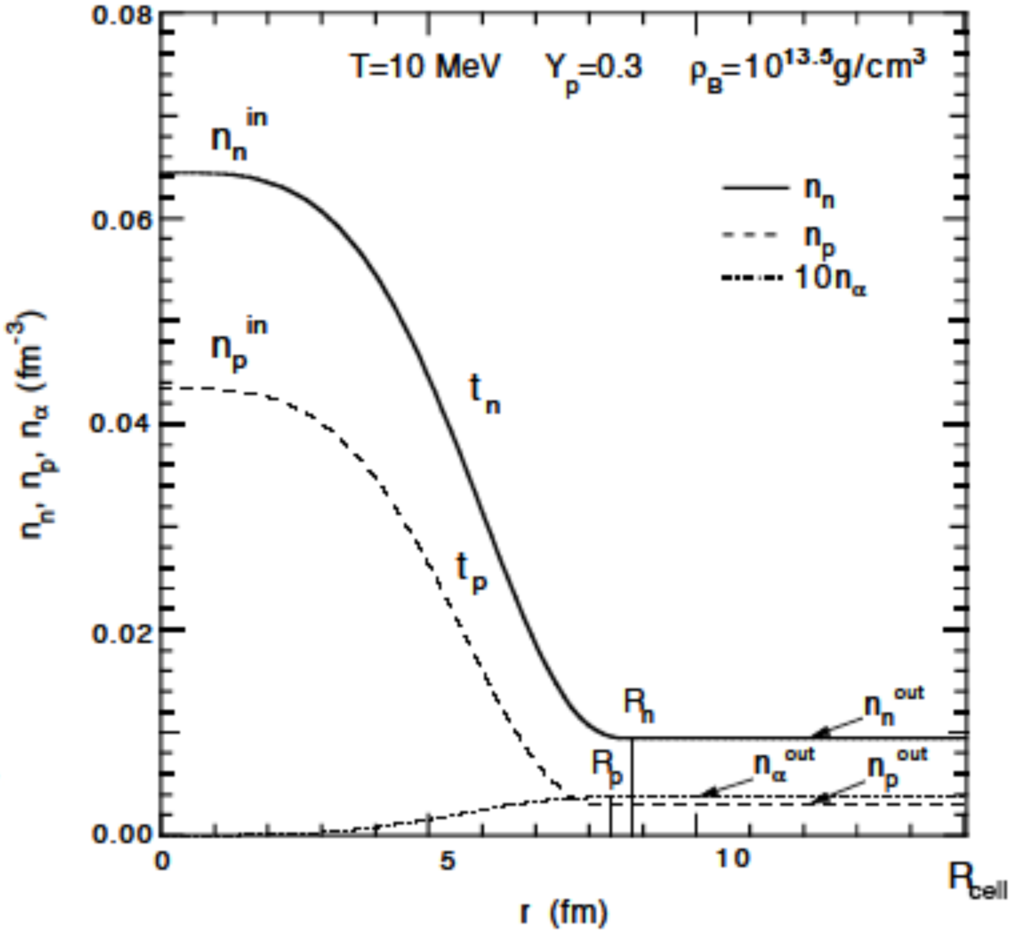
Charge neutral
BCC lattice



Minimize
Free energy density
In cell

$V_{\rm cell} = a^3 = N_B/n_B$

$\rho_B = m_u n_B$



- Non-uniform & Uniform
- Mix of: Neutron Proton Alpha Nucleus

$V_{\rm cell} = 4\pi R_{\rm cell}^3/3.$

$$n_i(r) = \begin{cases} \left(n_i^{\rm in} - n_i^{\rm out}\right)\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{\rm out}, & 0 \leqslant r \leqslant R_i, \\ n_i^{\rm out}, & R_i \leqslant r \leqslant R_{\rm cell}, \end{cases}$$

http://ecee.colorado.edu/~bart/book/bravais.htm#bcc

Shen et al. PTP (1998)

Local density approximation in cell

Free energy density: f

$$f = F_{\rm cell}/a^3 = (E_{\rm cell} - T S_{\rm cell})/a^3,$$

Free energy per cell: F

$$F_{\rm cell} = (E_{\rm bulk} + E_s + E_C) - TS_{\rm cell} = F_{\rm bulk} + E_s + E_C.$$

Bulk energy:

$$F_{\rm bulk} = \int_{\rm cell} f\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r.$$

$$E_{\rm bulk} = \int_{\rm cell} \epsilon\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r,$$

$$S_{\rm cell} = \int_{\rm cell} s\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r,$$

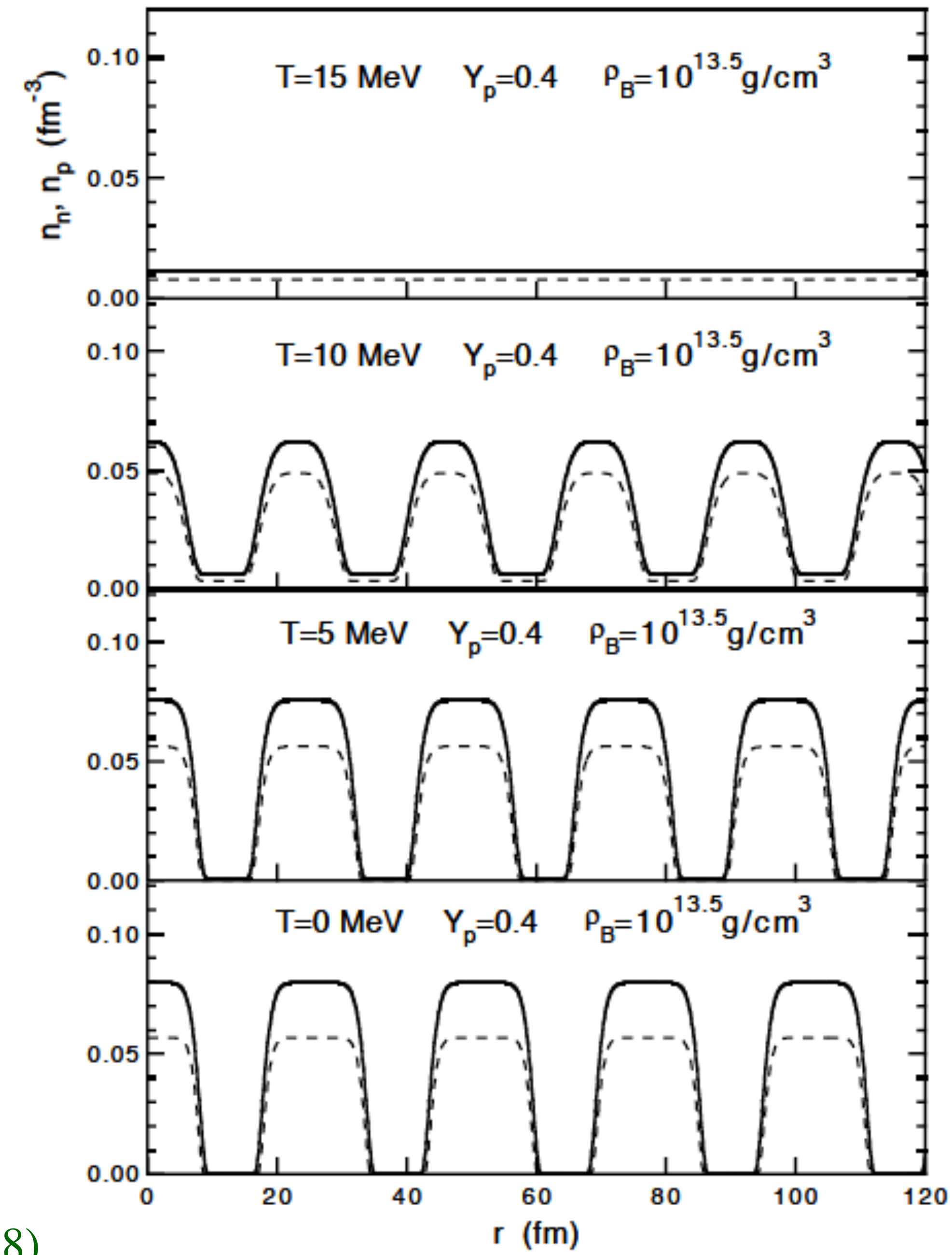
Surface energy:

$$E_s = \int_{\rm cell} F_0 \,|\, \nabla(n_n(r) + n_p(r)) \,|^2 \, d^3r.$$

Coulomb energy:

$$E_C = \frac{1}{2}\int_{\rm cell} e[n_p(r) + 2n_\alpha(r) - n_e]\,\phi(r) d^3r + \Delta E_C,$$

Local density approximation in cell



- 非一様分布
  &
  一様分布

- 混合物質
  (陽子, 中性子,
  ヘリウム,
  原子核)

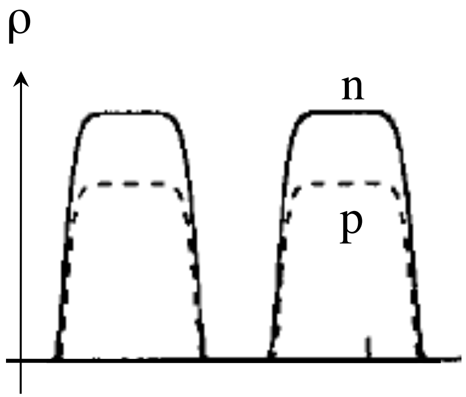
↑ temperature

温度が高くなると
原子核が溶けていく

Shen et al. PTP (1998)

Collapse

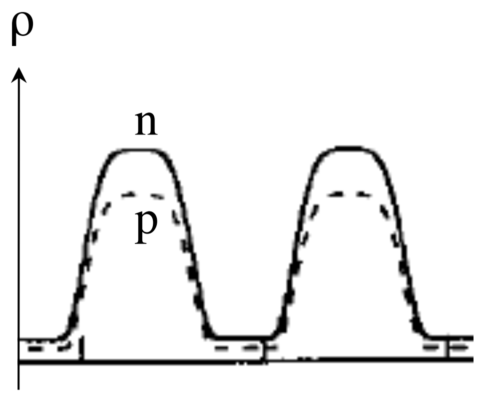
$\rho_c \sim 10^{10}$ g/cm^3
$T_c \sim 1$ MeV
$Y_e \sim 0.42$



Nuclei,
p, e^-

ν-trapping

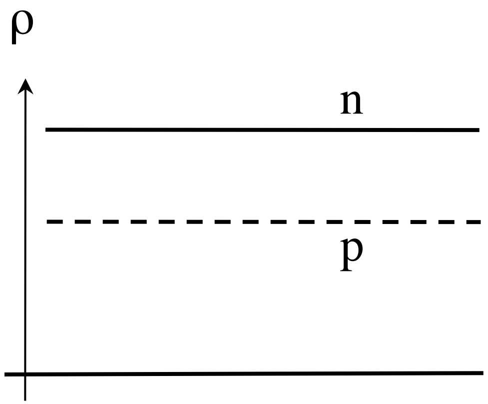
$\rho_c \sim 10^{12}$ g/cm^3
$T_c \sim 2$ MeV
$Y_e \sim 0.40$



Nuclei,
p, n, e^-, ν_e

Core-Bounce

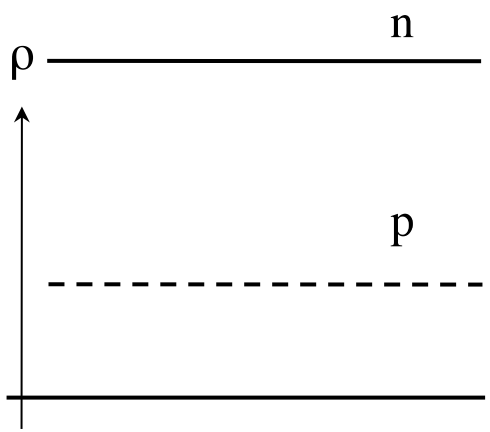
$\rho_c \sim 3 \times 10^{14}$ g/cm^3
$T_c \sim 10$ MeV
$Y_e \sim 0.30$



p, n,
e^-, ν_e

Explosion, Neutron stars

$\rho_c \sim 5 \times 10^{14}$ g/cm^3
$T_c \sim 15$ MeV
$Y_e < 0.20$



p, n, (Λ, q)
e^-, ν_i, $\overline{\nu}_i$

Shen EOS table for supernovae

H. Shen, Toki, Oyamatsu & Sumiyoshi NPA, PTP(1998), ApJS (2012)

- Covers wide range of
  - Density: $10^{5.1} \sim 10^{16}$ g/cm^3
  - Proton fraction: 0 ~ 0.65
  - Temperature: 0 ~ 400 MeV
- Data table ~140 MB (110 x 66 x 92 points)

Shen-EOS

  - Quantities: ε, p, S, μ_i, X_i, m*
- One of the standard EOS table
  - over 660 citations (408+221+33)
- Implementation to simulations
  - Interpolation from EOS table

cccccccccccccccccccccccccccccccccccccccc
Temperature= 1.000000E-01

5.100000E+00	7.581421E-11	-2.000000E+00	1.000000E-02	-1.524779E+00
5.200000E+00	9.544443E-11	-2.000000E+00	1.000000E-02	-1.502472E+00
5.300000E+00	1.201574E-10	-2.000000E+00	1.000000E-02	-1.480166E+00
5.400000E+00	1.512692E-10	-2.000000E+00	1.000000E-02	-1.457861E+00
5.500000E+00	1.904367E-10	-2.000000E+00	1.000000E-02	-1.435557E+00
5.600000E+00	2.397456E-10	-2.000000E+00	1.000000E-02	-1.413255E+00
5.700000E+00	3.018218E-10	-2.000000E+00	1.000000E-02	-1.390953E+00
5.800000E+00	3.799711E-10	-2.000000E+00	1.000000E-02	-1.368653E+00
5.900000E+00	4.783553E-10	-2.000000E+00	1.000000E-02	-1.346354E+00
6.000000E+00	6.022137E-10	-2.000000E+00	1.000000E-02	-1.324056E+00
6.100000E+00	7.581421E-10	-2.000000E+00	1.000000E-02	-1.301759E+00
6.200000E+00	9.544443E-10	-2.000000E+00	1.000000E-02	-1.279464E+00
6.300000E+00	1.201574E-09	-2.000000E+00	1.000000E-02	-1.257169E+00
6.400000E+00	1.512692E-09	-2.000000E+00	1.000000E-02	-1.234876E+00
6.500000E+00	1.904367E-09	-2.000000E+00	1.000000E-02	-1.212584E+00
6.600000E+00	2.397456E-09	-2.000000E+00	1.000000E-02	-1.190294E+00

http://user.numazu-ct.ac.jp/~sumi/eos

Comparison of EOSs (1)

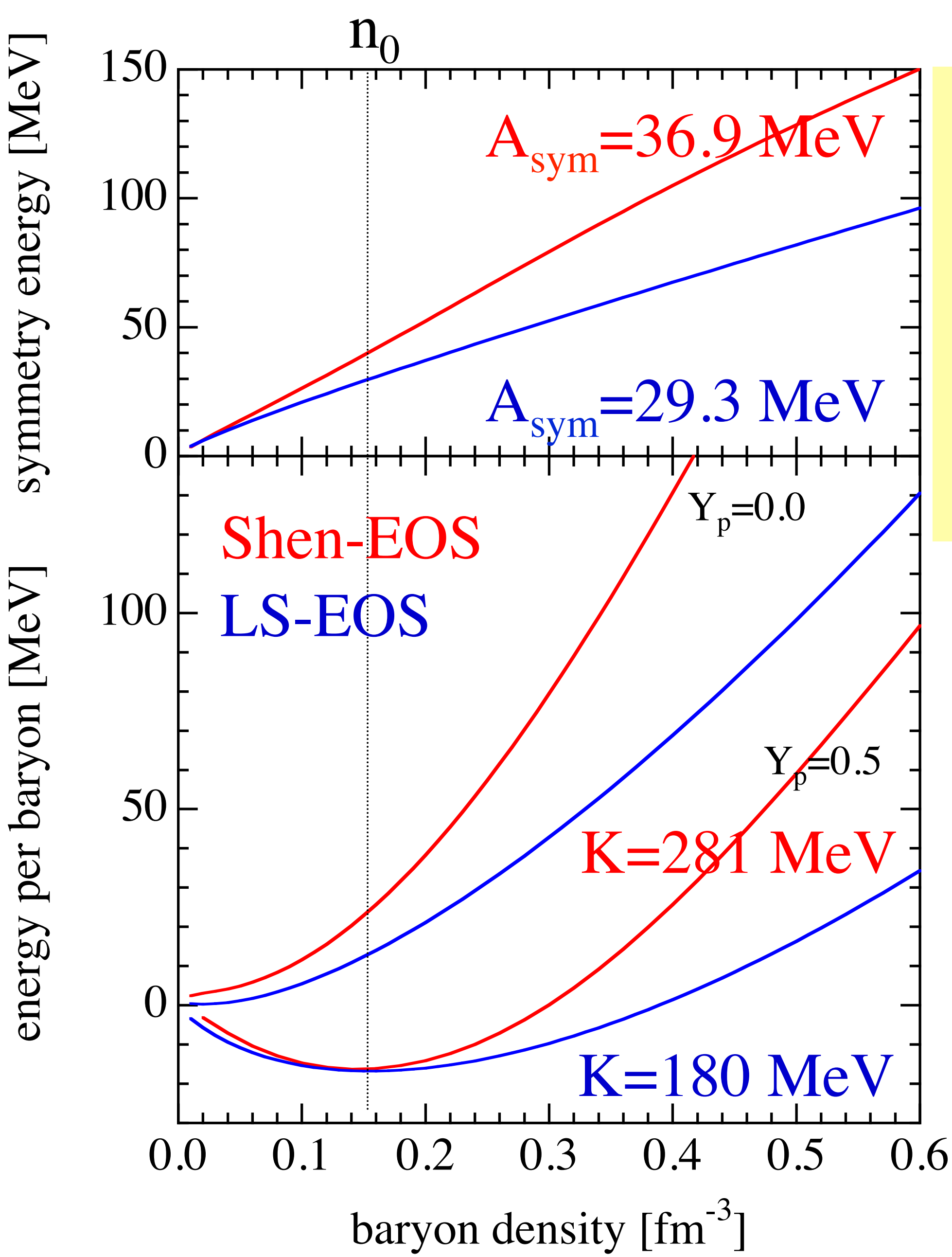
	LS-EOS	Shen-EOS
Model	Compressible liquid drop model	Rel. Mean Field + Local-Density Approx.
Bulk EOS	“Skyrme”-like	RMF (RBHF)
Interaction	Saturation	Mass, R_c, R_n
Nucl. Data	partly	Yes (incl. unstable)
n-skin	---	Yes
M*	---	Yes
Info	Subroutine	Data table

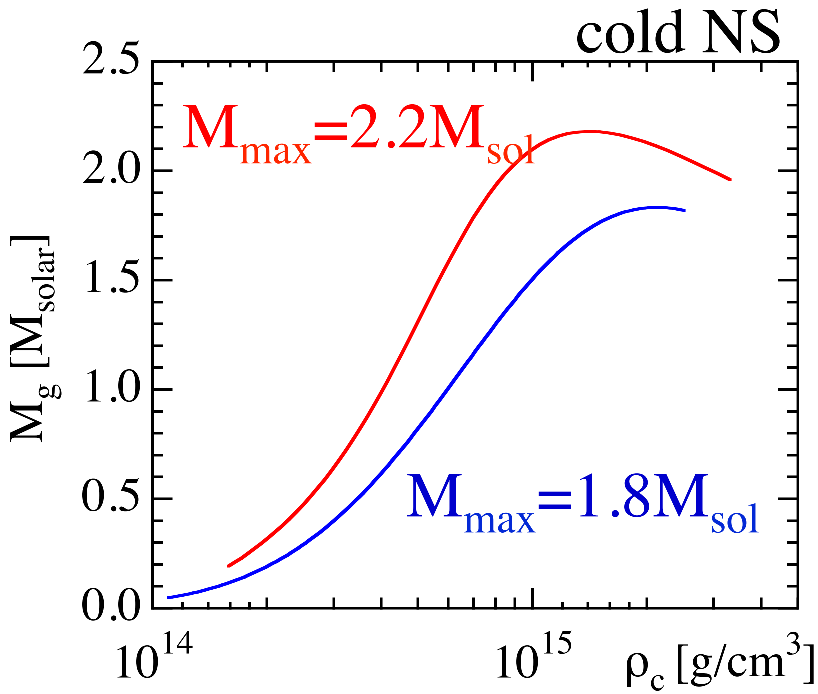
Comparison of EOSs (2)

	LS-EOS	Shen-EOS
K [MeV]	180, 220, 375	281
A_{sym} [MeV]	29.3	36.9
Max. NS mass [M_{sol}]	1.8, 2.0, 2.7	2.2

Shen-EOS vs LS-EOS

- 堅い状態方程式 cf. non-rel
  - コアバウンスへ影響
  - 中性子星の性質
- 対称エネルギー効果が大きい
  - 不安定核で検証済み
  - 組成の違いへ影響
    - e-capture, ν-reaction rates





Sumiyoshi et al. NPA730 (2004)