格子QCDによるハドロン間相互作用

-核力をQCDから導く-

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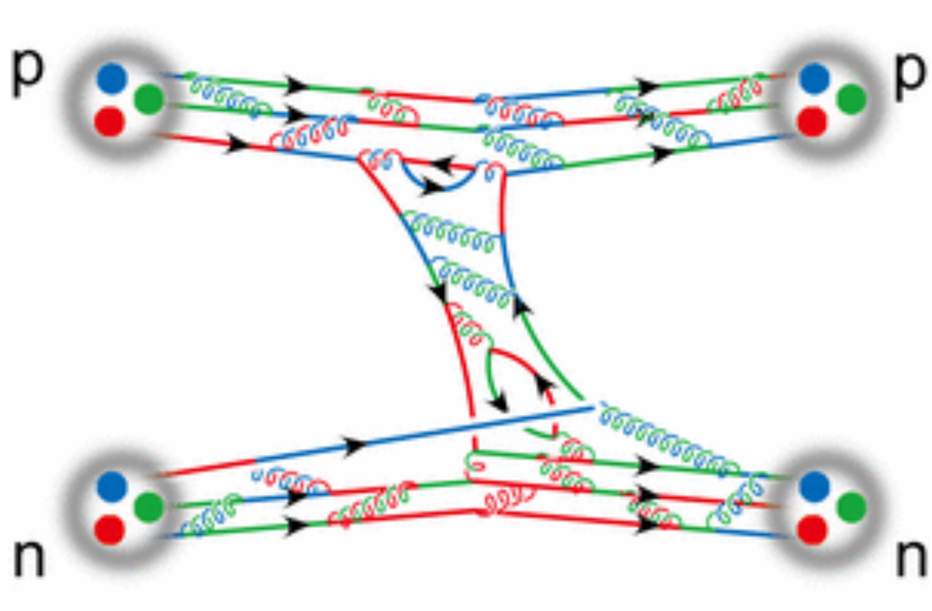
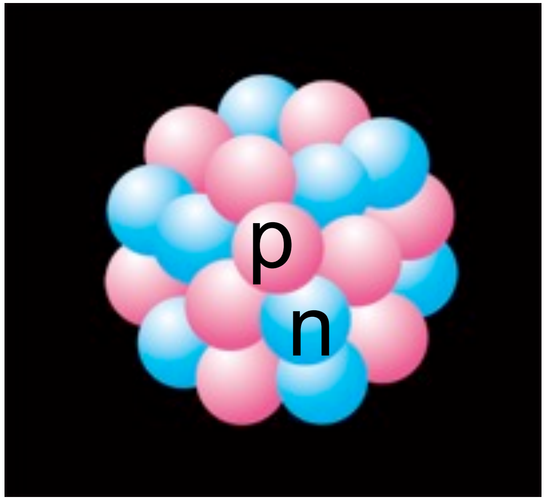


京都大学基礎物理学研究所セミナー

2010.12.16

1. Motivation

What binds protons and neutrons inside a nuclei ?



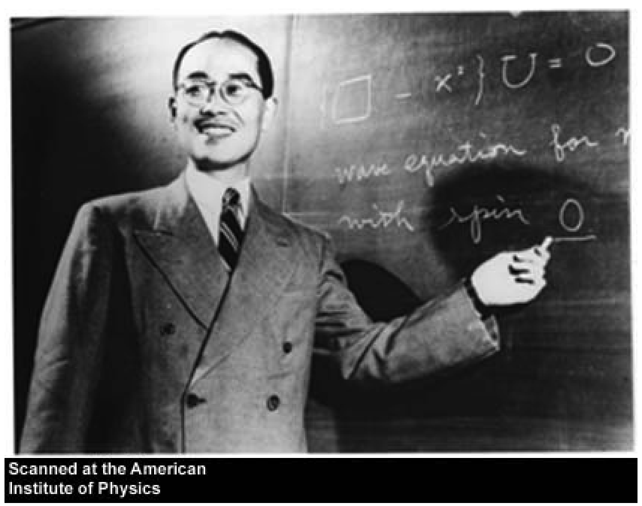
gravity: too weak

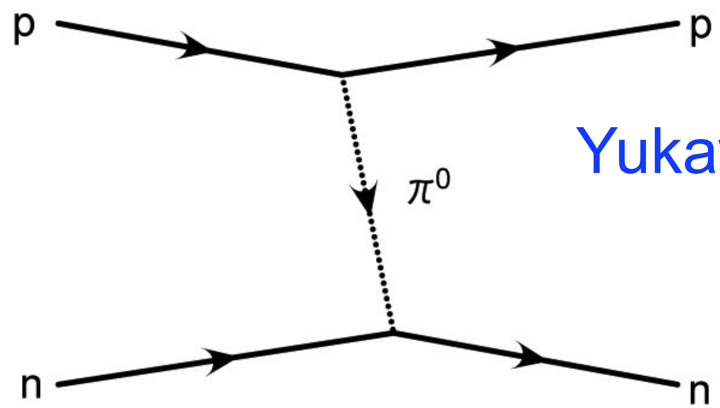
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force





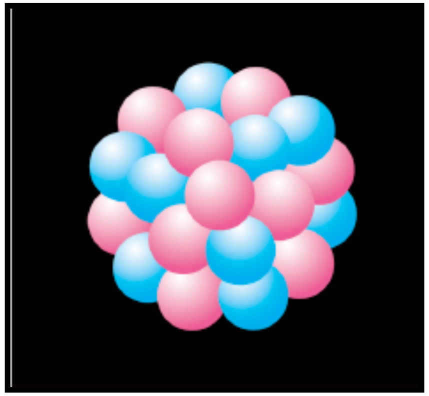
Yukawa potential

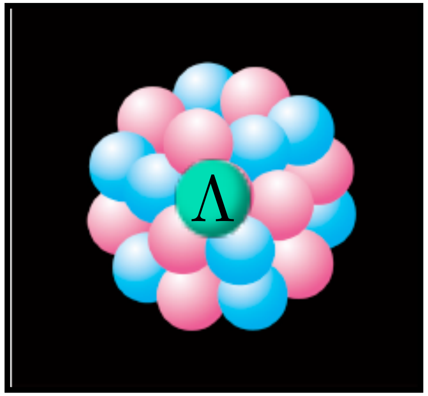
$$V(r) = \frac{g^2}{4\pi} \frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

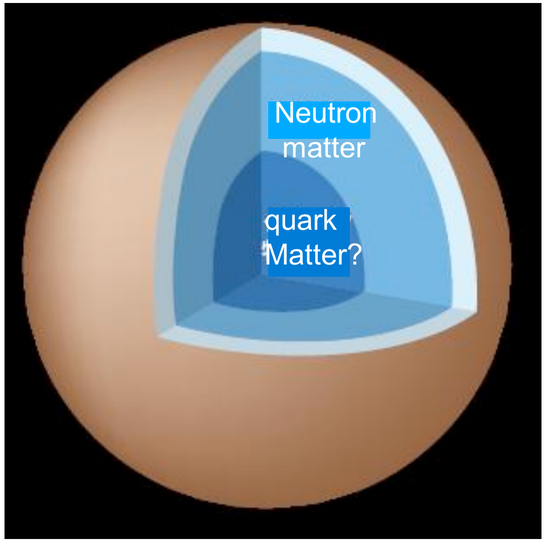
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei





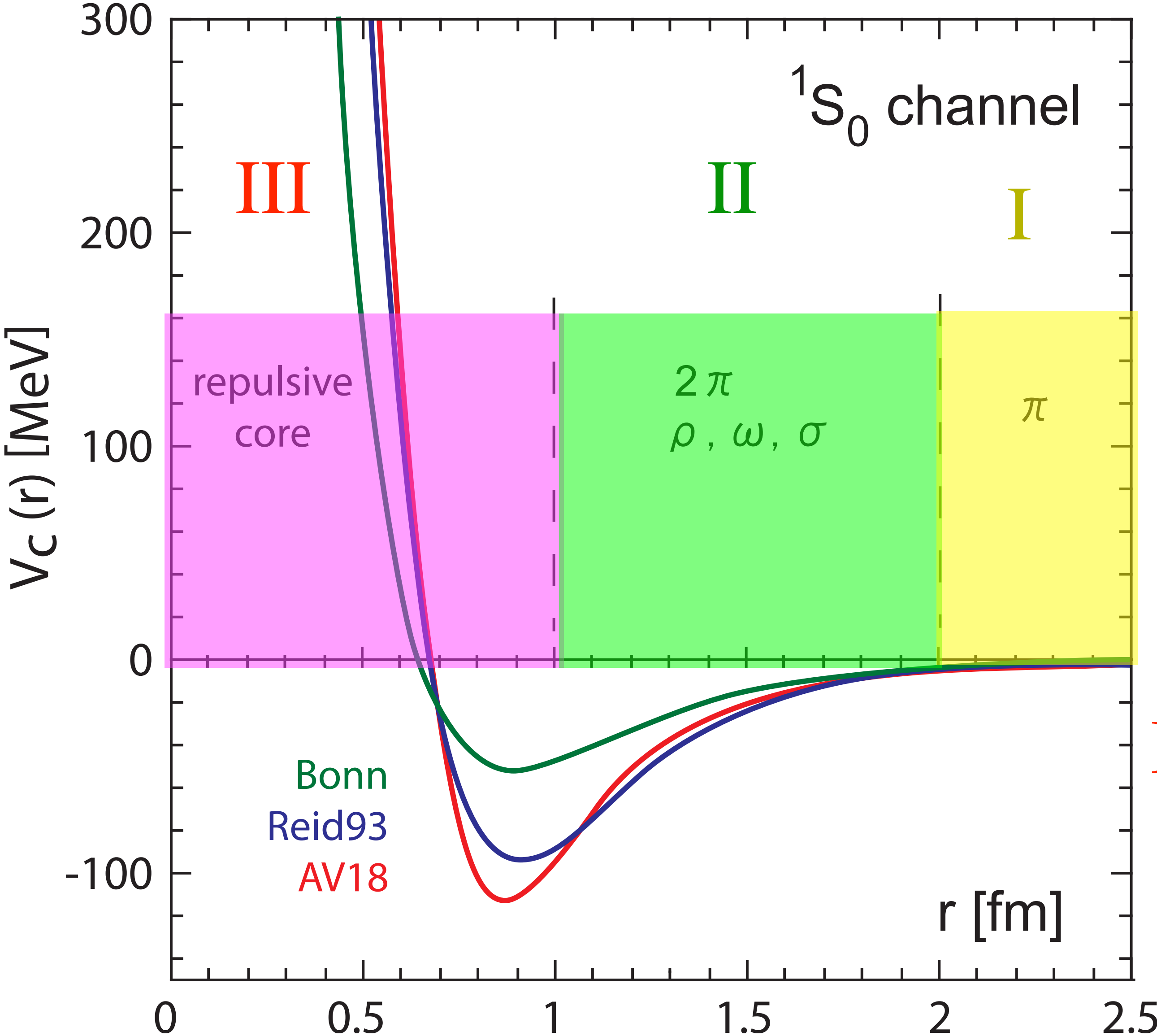
- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

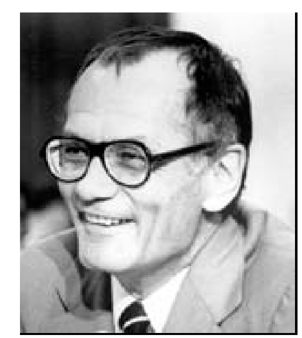
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)

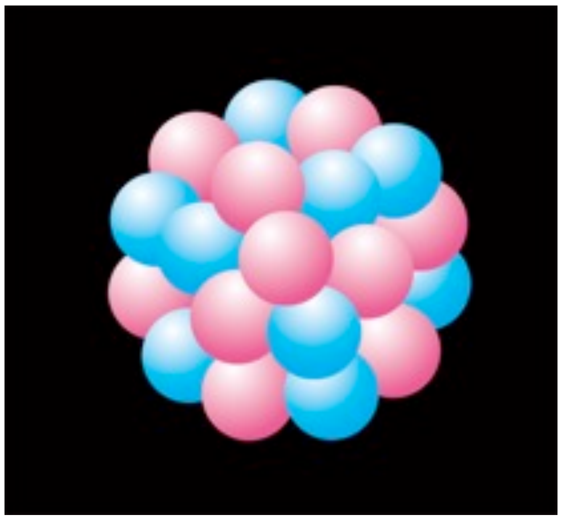


Repulsive core is important

stability of nuclei

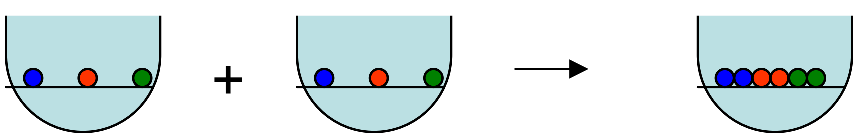
maximum mass of neutron star

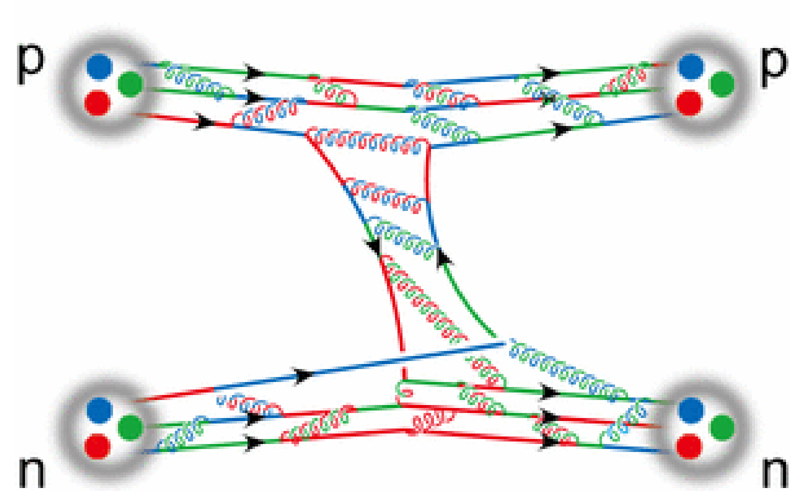
explosion of type II supernova



Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.





核力の性質をクォークから説明できるか？

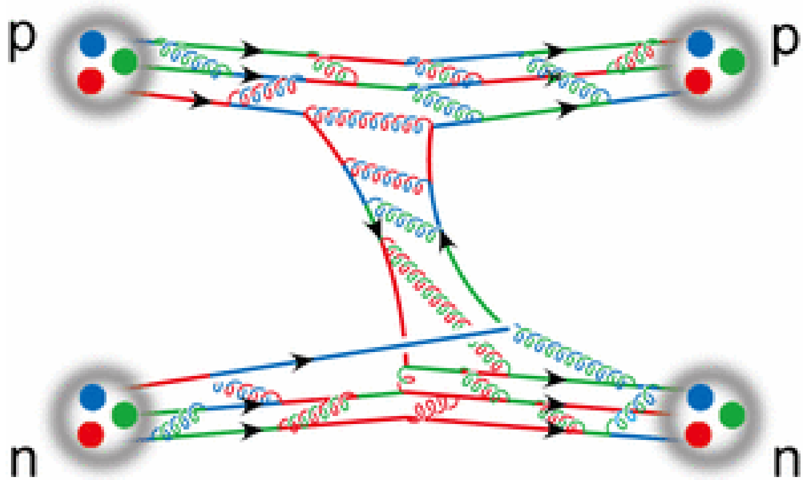
Plan of my talk

1. Motivation
2. Strategy in (lattice) QCD to extract “potential”
3. More structure: tensor potential
4. Inelastic scattering: octet baryon interactions
    1. Baryon-Baryon interactions in an SU(3) symmetric world
    2. Proposal for S=-2 inelastic scattering
    3. H-dibaryon
5. Summary and Discussion

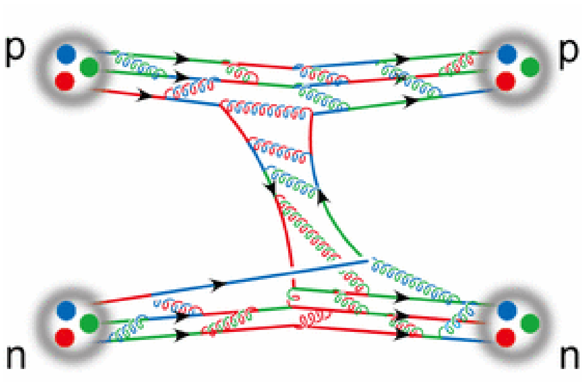
2. Strategy in (lattice) QCD to extract “potential”

南部陽一郎、『クォーク』第２版（講談社、ブルーバックス、1997）

『現在でも核力の詳細を基本方程式から導くことはできない。核子自体がもう素粒子とは見なされないから、いわば複雑な高分子の性質をシュレーディンガー方程式から出発して決定せよというようなもので、むしろこれは無理な話である。』



QCDから核力を如何に定義し、如何に計算するか？



Y. Nambu,
"Force Potentials in Quantum Field Theory",
Prog. Theor. Phys. 5 (1950) 614.

C. Hayashi and Y. Munakata,
"On a Relativistic Integral Equation for Bound states", Prog. Theor. Phys. 7 (1952) 481.

K. Nishijima,
"Formulation of Field Theories for composite particles", Phys. Rev. 111 (1958) 995.

HAL QCD Collaboration

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池田陽一 (理研)、井上貴史 (日大)
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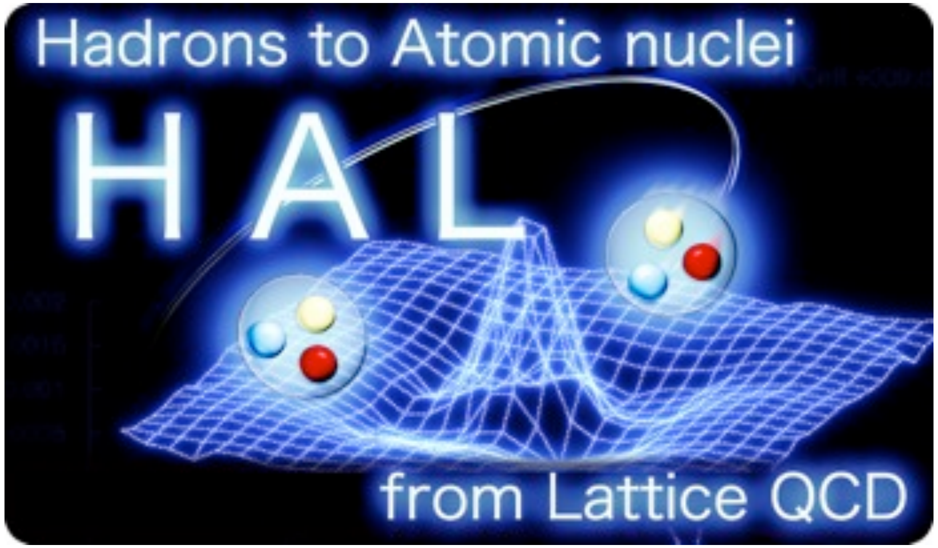


Ishii-Aoki-Hatsuda, PRL 90(2007)0022001
Aoki-Hatsuda-Ishii, PTP123(2010)89

(equal time) Nambu-Bethe-Salpeter wave function is a key

$$\varphi_E(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|6q,E\rangle$$

$E = 2\sqrt{\mathbf{k}^2 + m_N^2}$

QCD eigen-state with energy E and #quark =6

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

$E < E_{th}$

C.-J.D.Lin et al., NPB69(2001) 467
CP-PACS Coll., PRD71 (2005) 094504
N. Ishizuka, PoS(LAT2009)119

off-shell T-matrix

$$\varphi_E(\mathbf{r}) = C\left[e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p},-\mathbf{p}\leftarrow\mathbf{k},-\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon} + \mathcal{I}(\mathbf{r})\right]$$

inelastic contribution $\propto O(e^{-\sqrt{E_{th}^2 - E^2}|\mathbf{r}|})$

(Relativistic) Spinor structure is contained in C. N. Ishizuka, PoS(LAT2009)119

(Equal time) contains sufficient information.

同時刻、 非重心系 (空間的) 非同時刻、 重心系

$\mathbf{p} + \mathbf{q}$ Lorentz transformation $\mathbf{k} + (-\mathbf{k})$

Asymptotic behavior $r = |\mathbf{r}| \to \infty$

$$\varphi_E(\mathbf{r}) \longrightarrow \sum_l C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

partial wave

spinor structure

$\delta_l(k)$ is the scattering phase shift

$$S = e^{2i\delta}$$

S-matrix below inelastic threshold

Our definition of "potential"

Ishii-Aoki-Hatsuda, PRL 90(2007)0022001
Aoki-Hatsuda-Ishii, PTP123(2010)89

$$[\epsilon_k - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu}$$

$$H_0 = \frac{-\nabla^2}{2\mu}$$

$U(\mathbf{x},\mathbf{y})$ may be non-local but can be energy-independent.

$$\tilde{\varphi}_E(y) \qquad \langle\tilde{\varphi}_E|\varphi_{E'}\rangle = \delta_{EE'} \qquad \sum_{E\le E_{\rm th}} |\varphi_E\rangle\langle\tilde{\varphi}_E| = \mathbf{1}_{E\le E_{\rm th}}$$

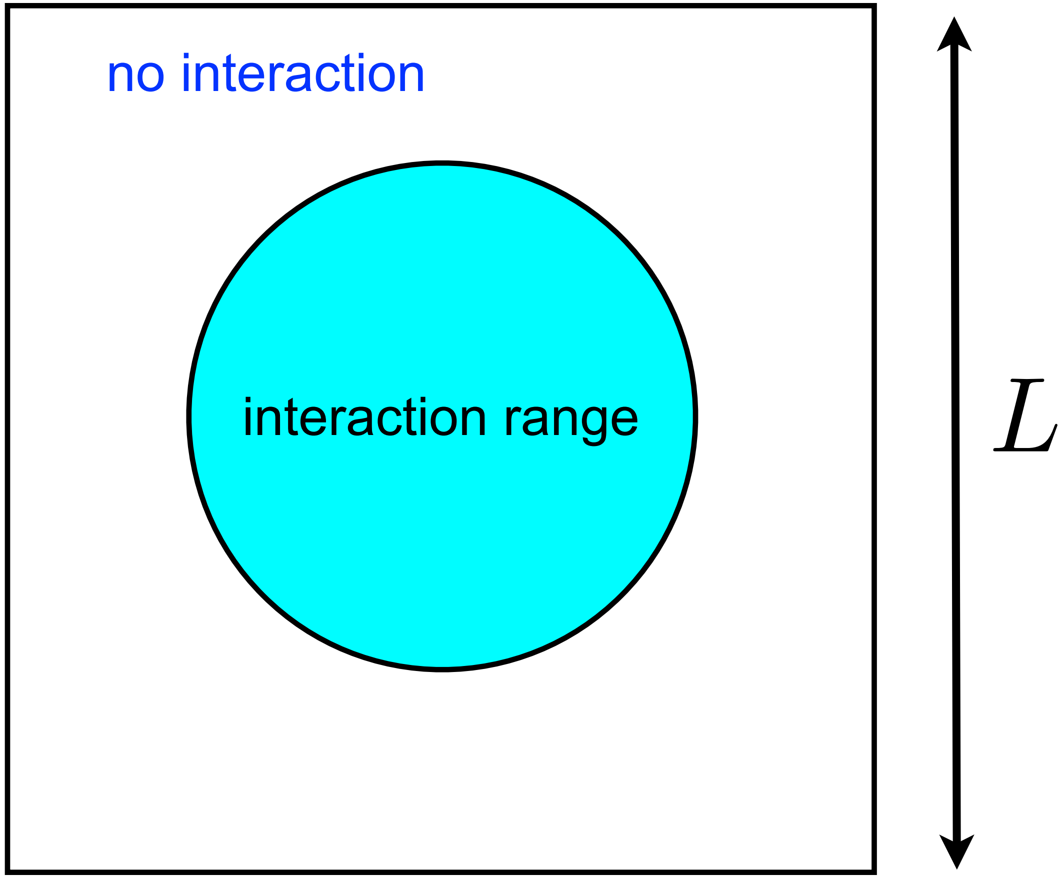
dual basis

identity in the restricted space

$$U(\mathbf{x},\mathbf{y}) = \sum_{E\le E_{\rm th}} [\varepsilon_k - H_0]\varphi_E(\mathbf{x})\tilde{\varphi}_E(\mathbf{y})$$

this construction is NOT unique.

Finite but large volume



Finite volume

↓

allowed value: k_n^2

↓

Lueshcer's formula

$\delta_l(k_n)$

"potential" is expected to be short-range.

Velocity expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

Okubo-Marshak (1958)

spins

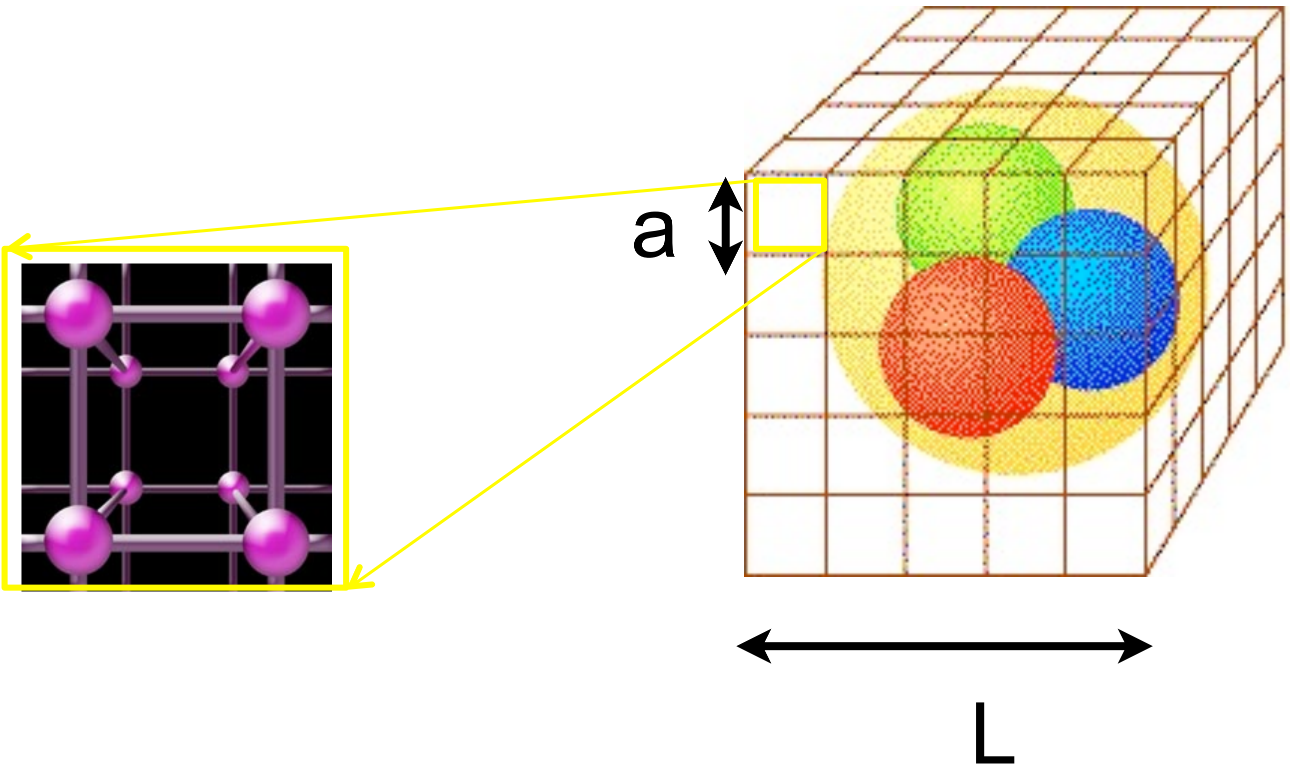
$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$

we calculate observables such as the phase shift and the binding energy, using this approximated potential.

3. Results from lattice QCD



- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

NBS wave function from lattice QCD

$$\langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)\overline{\mathcal{J}}_{pn}(t_0)|0\rangle = \langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)\sum_n |E_n\rangle\langle E_n|\overline{\mathcal{J}}_{pn}(t_0)|0\rangle$$

$$= \sum_n A_n \langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)|E_n\rangle e^{-E_n(t-t_0)} \longrightarrow A_0 \varphi^{E_0}_{\alpha\beta}(\mathbf{x}-\mathbf{y}) e^{-E_0(t-t_0)}$$

$$t \to \infty$$

$$A_n = \langle E_n|\overline{\mathcal{J}}_{pn}(t_0)|0\rangle$$

Wall source $\overline{\mathcal{J}}_{pn}(t_0) = p^{\mathrm{wall}}(t_0) n^{\mathrm{wall}}(t_0)$

$$q(\mathbf{x},t_0) \to q^{\mathrm{wall}}(t_0) = \sum_{\mathbf{x}} q(\mathbf{x},t_0)$$

with Coulomb gauge fixing

$L = 0$ $\quad P = +$

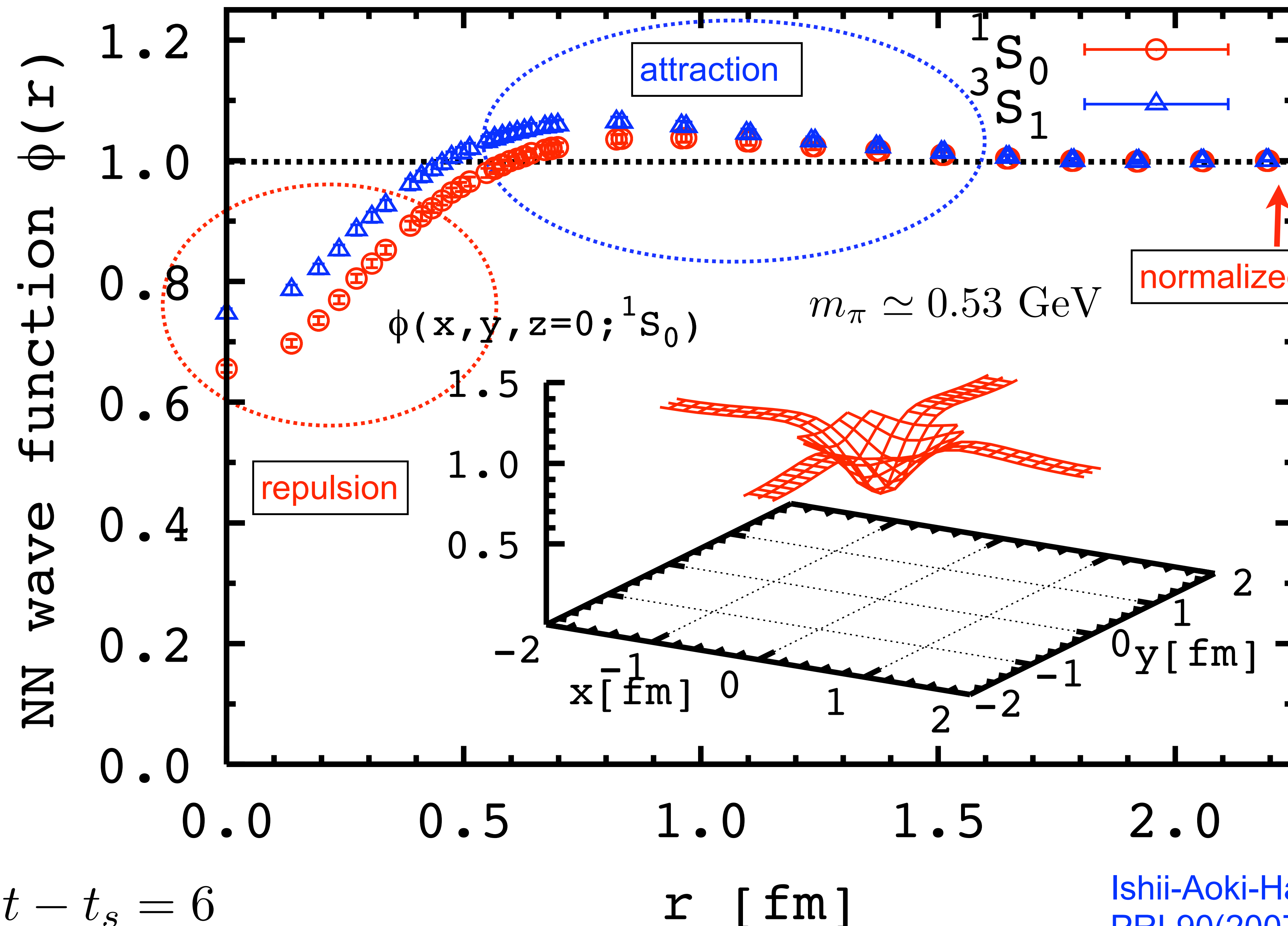
spin $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$^{2S+1}L_J$ ⇒ $^3S_1$ $\quad$ $^1S_0$

NN wave function

Quenched QCD

a=0.137fm



$t - t_s = 6$

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

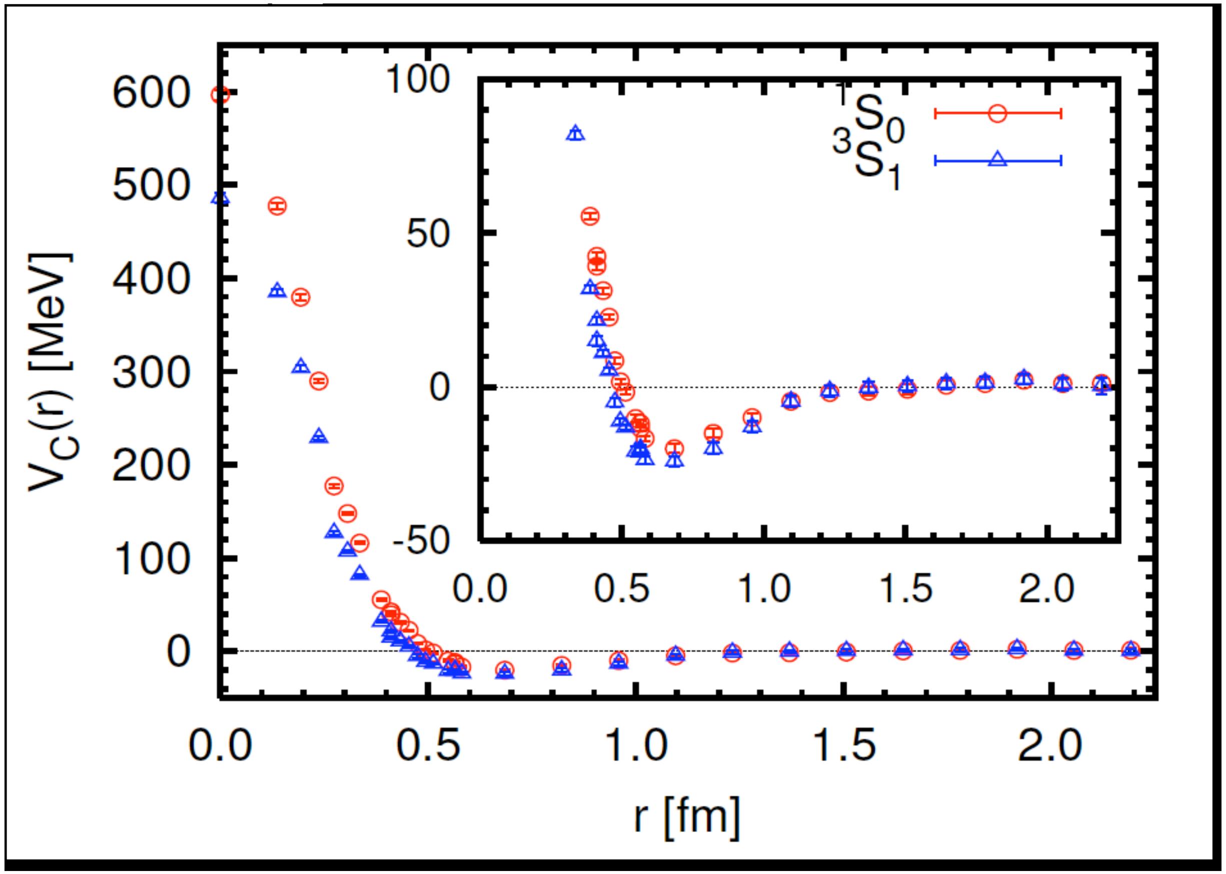
(quenched) potentials

LO (effective) central Potential

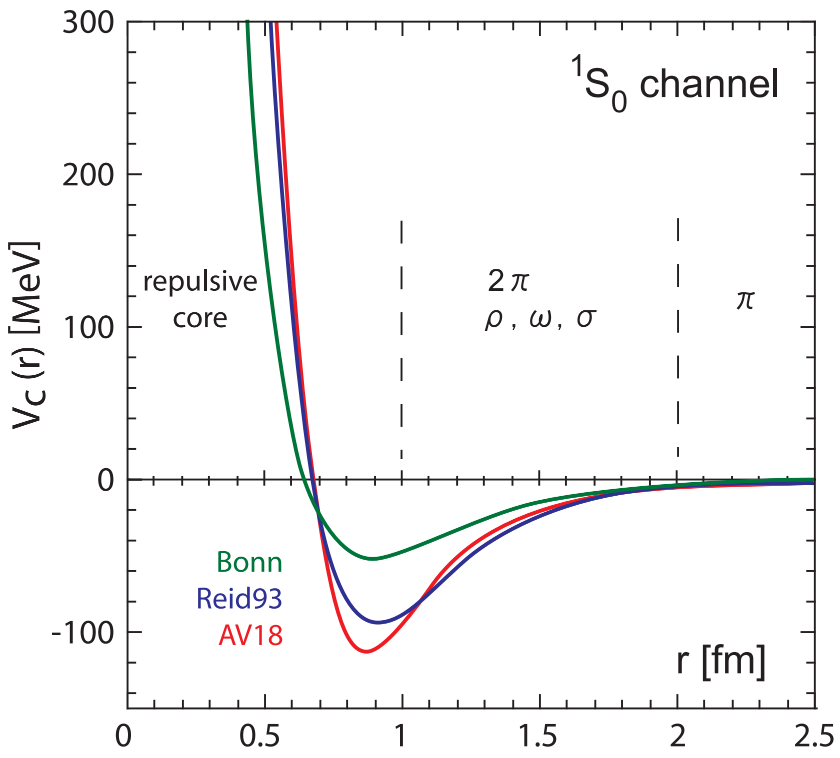
$$V(r;{}^1S_0) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r)$$

$$V(r;{}^3S_1) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r)$$

$E \simeq 0$ $\quad m_\pi \simeq 0.53$ GeV



a=0.137 fm L=4.4fm



Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007

Scheme/Operator dependence of “potential”

- The “potential” depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  - local operator = convenient choice for reduction formula
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron
- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - a similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
  - “good” scheme ?
    - good convergence of the perturbative expansion for the running coupling.
    - good convergence of the derivative expansion for the “potential” ?
      - completely local and energy-independent one is the best and must be unique if exists. (Inverse scattering method)

tools	running coupling	potential
physical observable	deep inelastic scattering	NN scattering phase shift
phenomena	almost free parton	repulsive core
interpretation	asymptotic freedom	no theoretical explanation so far
scheme	MS-bar coupling	potential from BS wave function

Other examples:

QM: (wave function,potential) $\rightarrow$ observables
QFT: (asymptotic field,vertex) $\rightarrow$ observables
EFT: (choice of field, vertex) $\rightarrow$ observables

Convergence of the derivative expansion

Leading Order

$$V_C(r) = \frac{(E - H_0)\varphi_E(\mathbf{x})}{\varphi_E(\mathbf{x})}$$

Local potential approximation

The local potential obtained at given energy E may depend on E.

Non-locality can be determined order by order in velocity expansion (cf. ChPT)

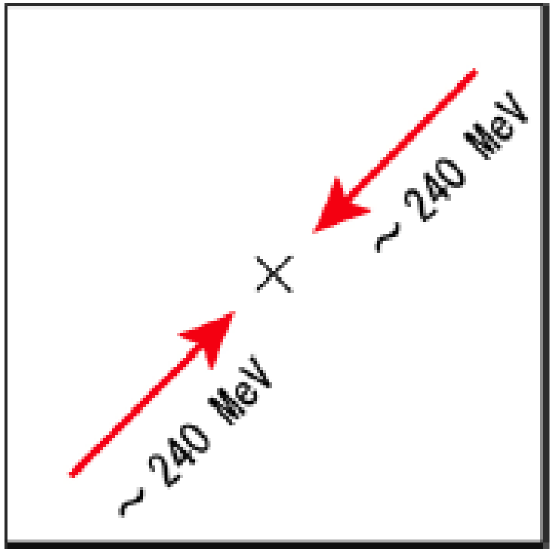
$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

Numerical check in quenched QCD

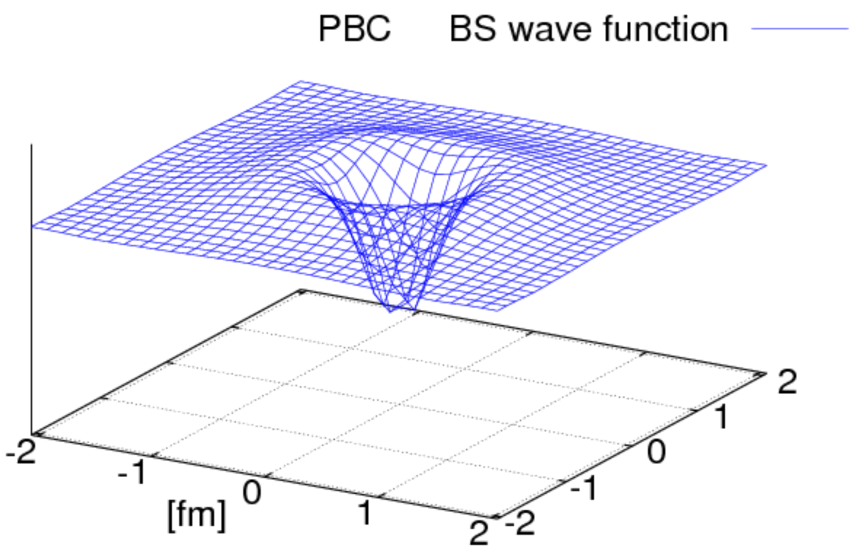
$m_\pi \simeq 0.53\ \mathrm{GeV}$

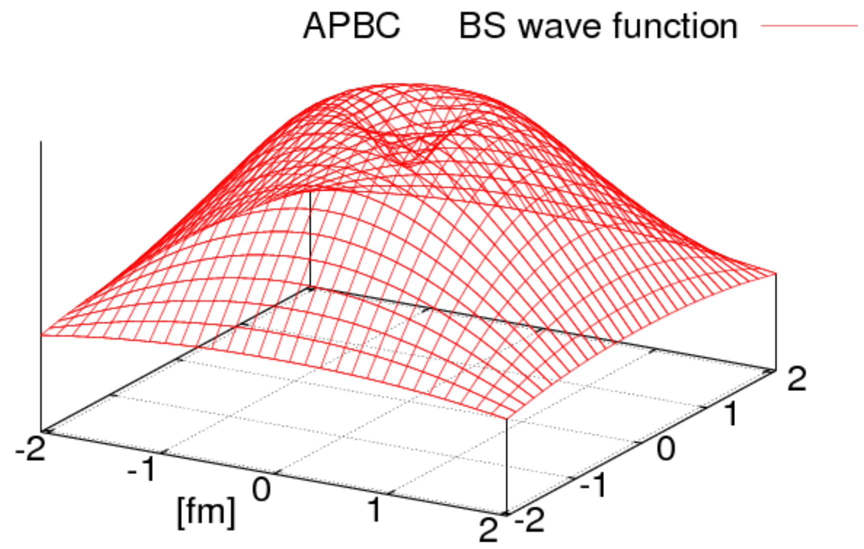
a=0.137fm

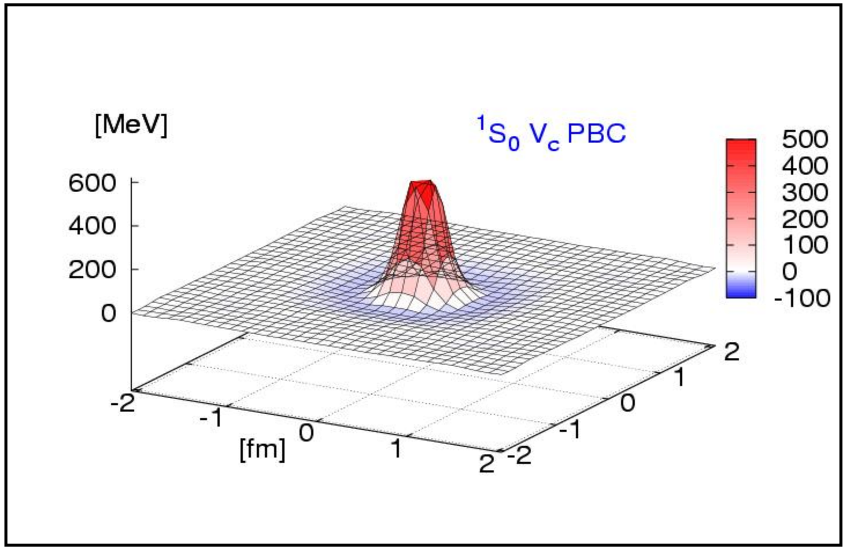
K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura

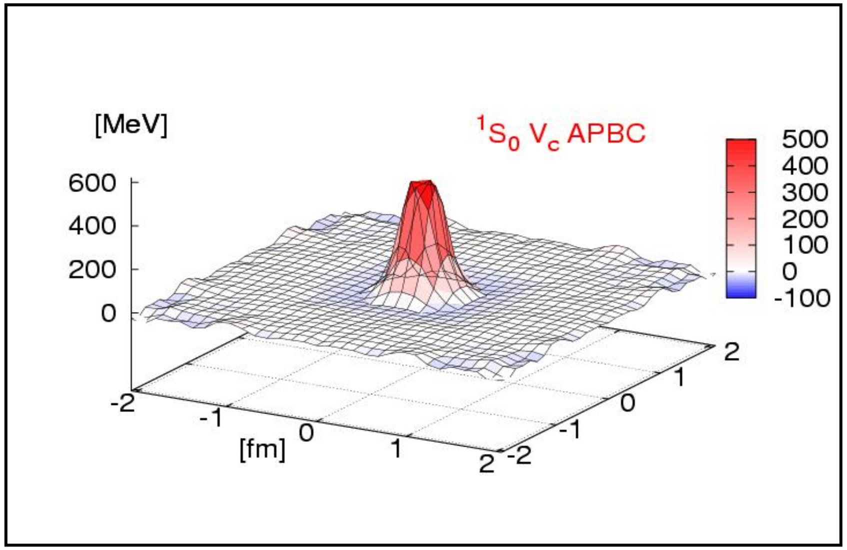


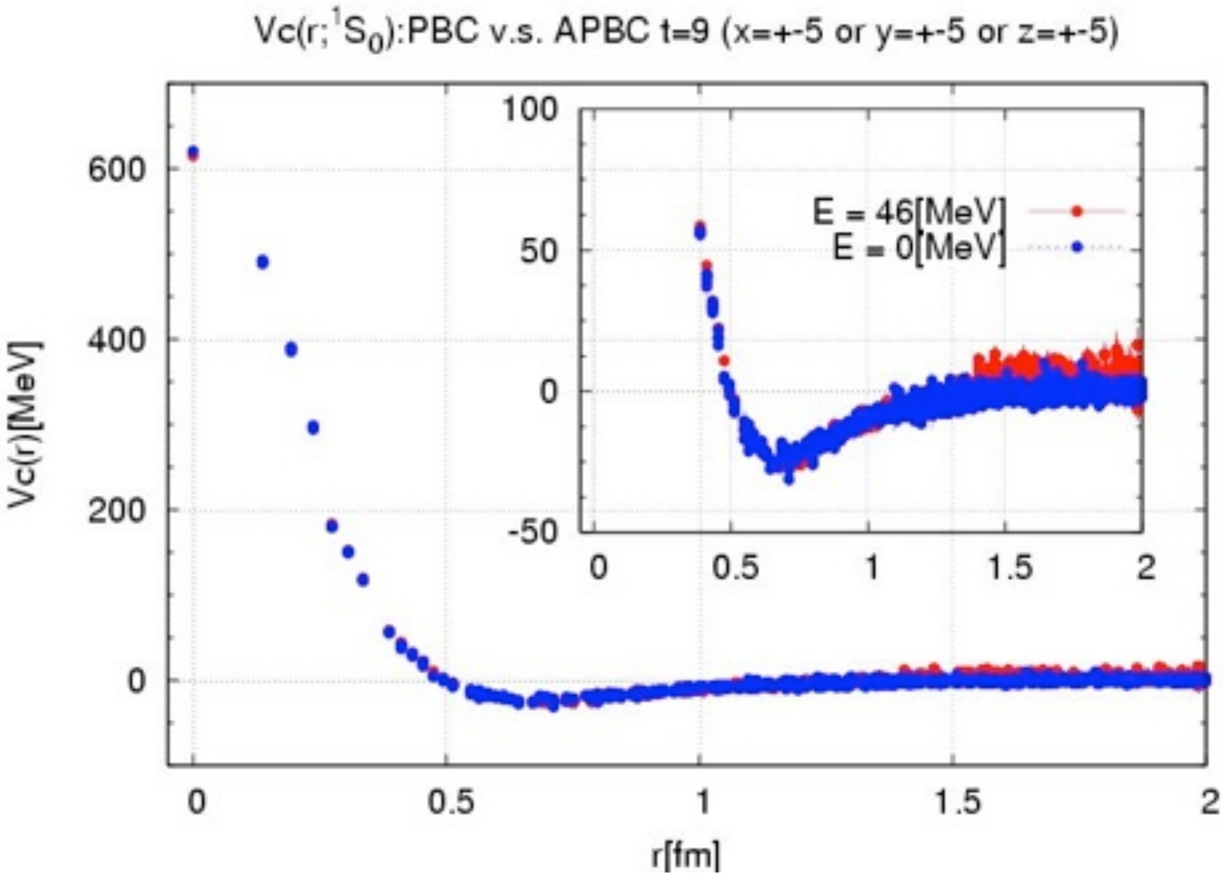
● PBC (E～0 MeV) ● APBC (E～46 MeV)









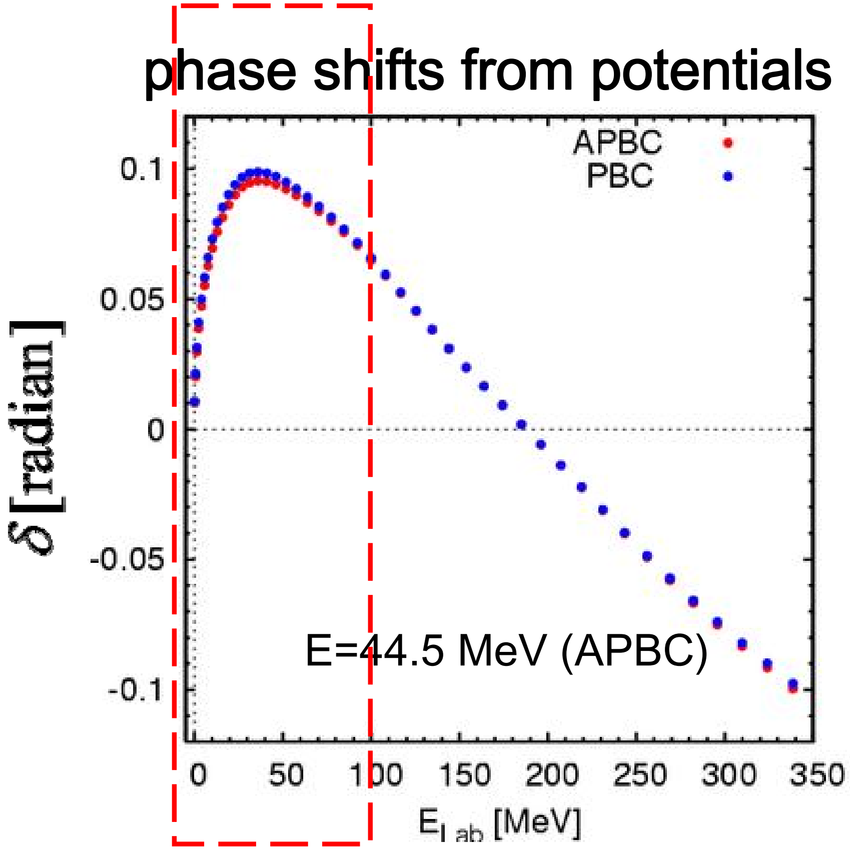


Quenched QCD

$m_\pi \simeq 0.53$ GeV

a=0.137fm

phase shifts from potentials



E-dependence of the local potential turns out to be very small at low energy in our choice of wave function.

Tensor potential

$$(H_0 + V_C + V_T S_{12})|\phi\rangle = E|\phi\rangle$$

mixing between $^3S_1$ and $^3D_1$ through the tensor force

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

$$|\phi_S\rangle = P|\phi\rangle = \frac{1}{24}\sum_{R\in\mathcal{O}} R|\phi\rangle$$ "projection" to L=0 $^3S_1$

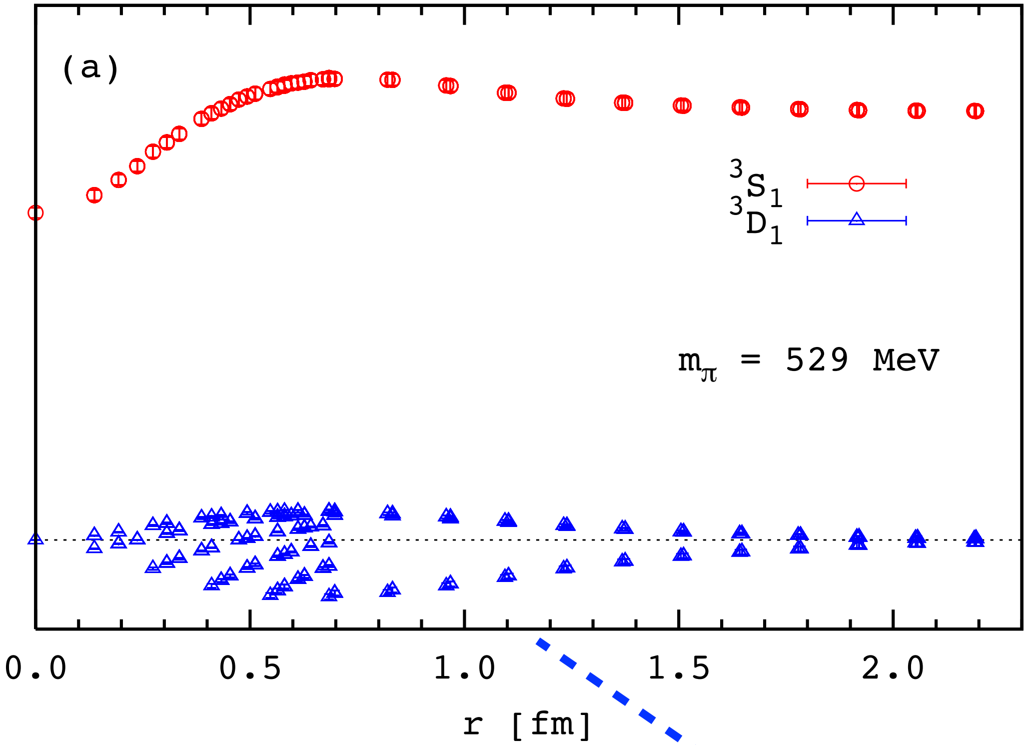
$$|\phi_D\rangle = Q|\phi\rangle = (1-P)|\phi\rangle$$ "projection" to L=2 $^3D_1$

$$\begin{aligned} P(H_0 + V_C + V_T S_{12})|\phi\rangle &= EP|\phi\rangle \\ Q(H_0 + V_C + V_T S_{12})|\phi\rangle &= EQ|\phi\rangle \end{aligned}$$

Wave functions

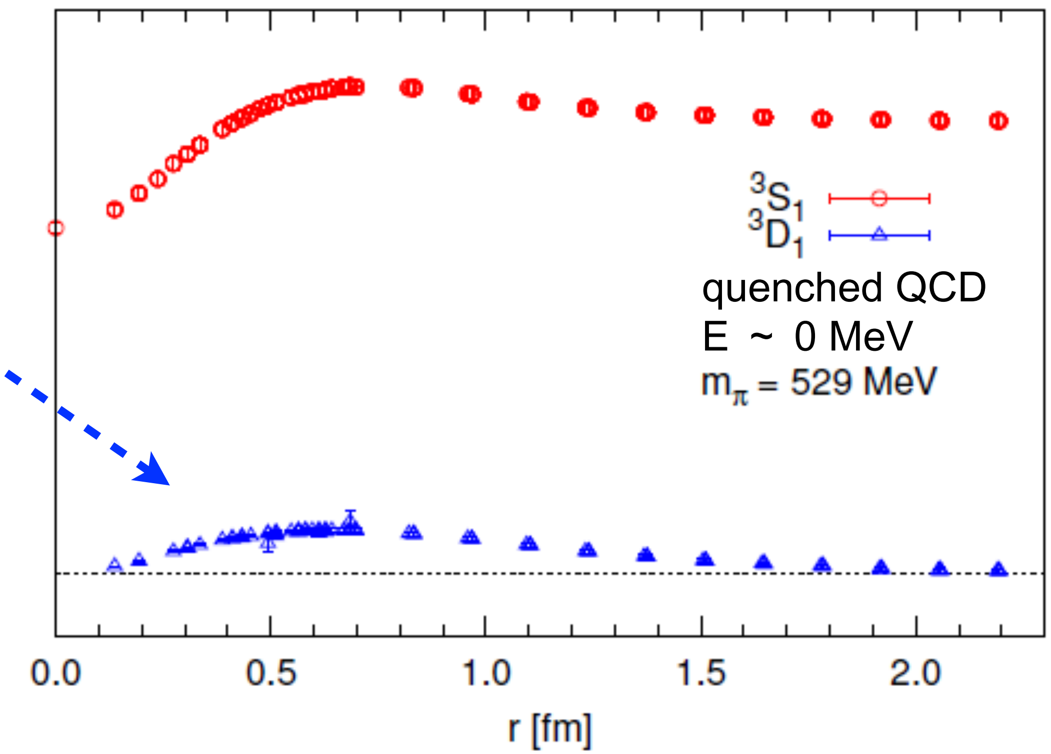
Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



remove angular dependence

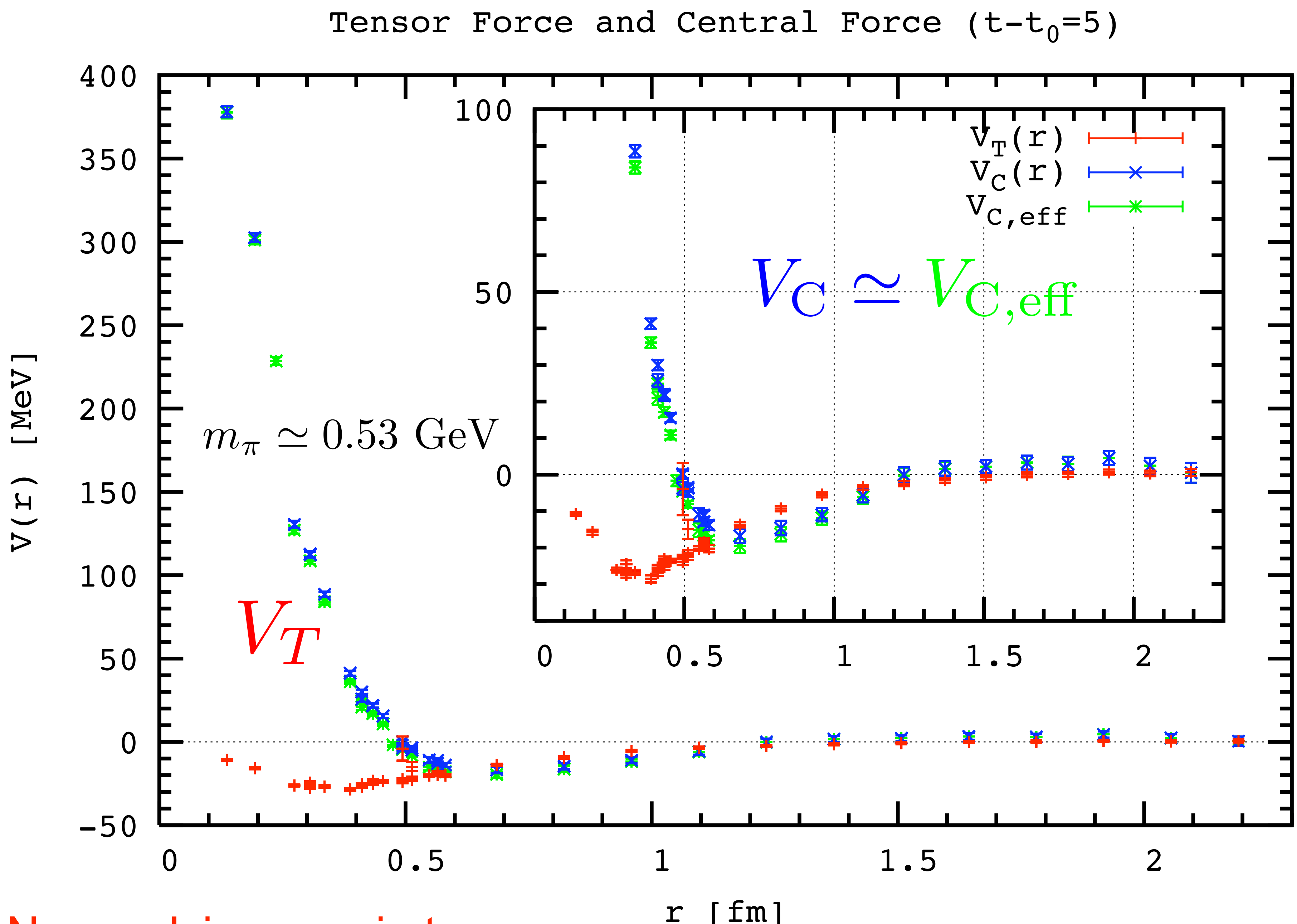
$$Y_{20}(\theta, \phi) \propto 3\cos^2\theta - 1$$



Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

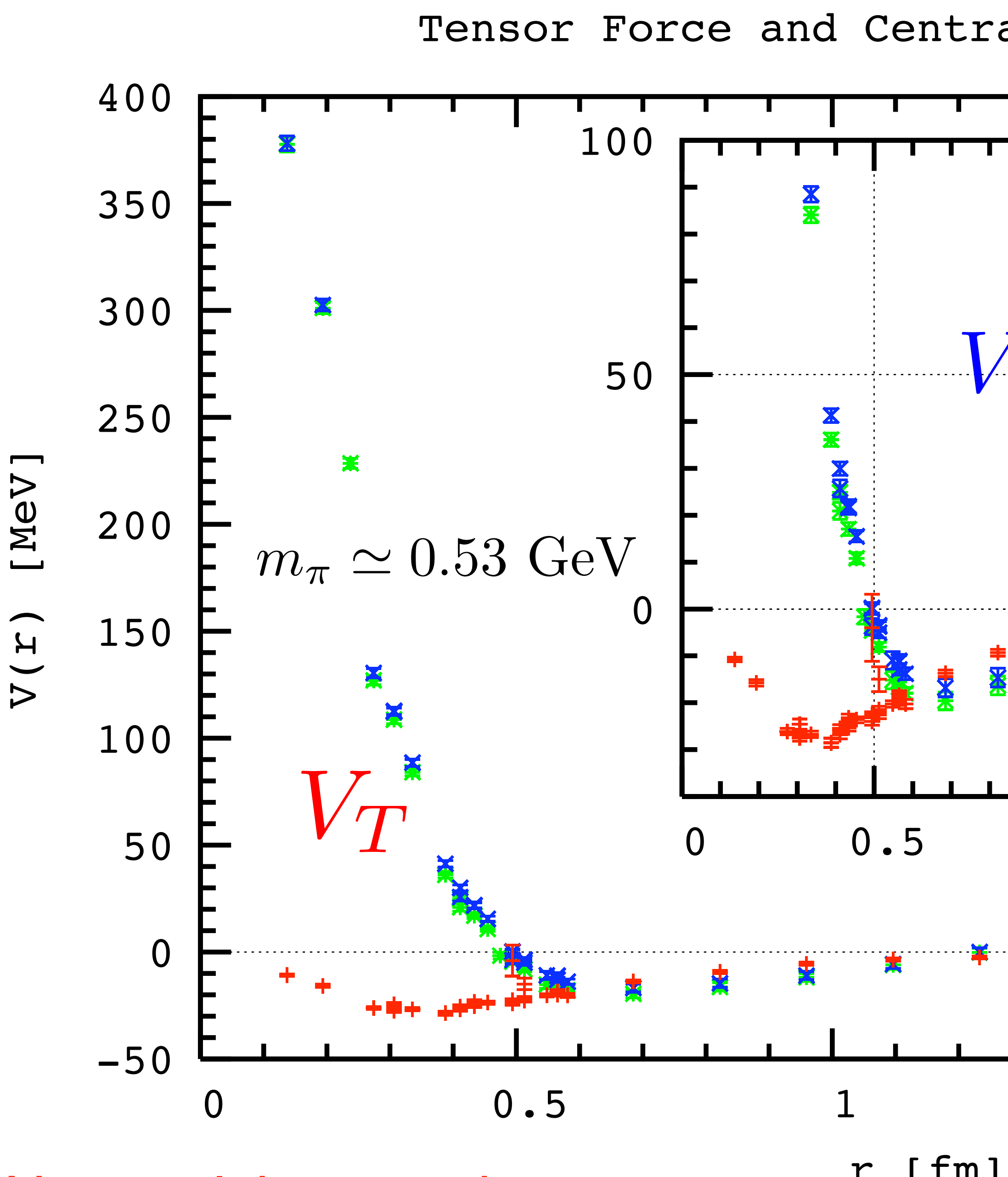


No repulsive core in tensor

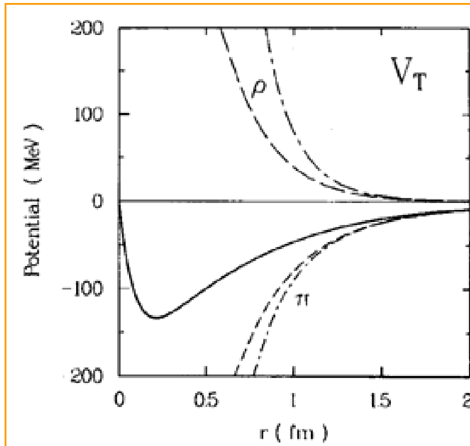
Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



No repulsive core in tensor

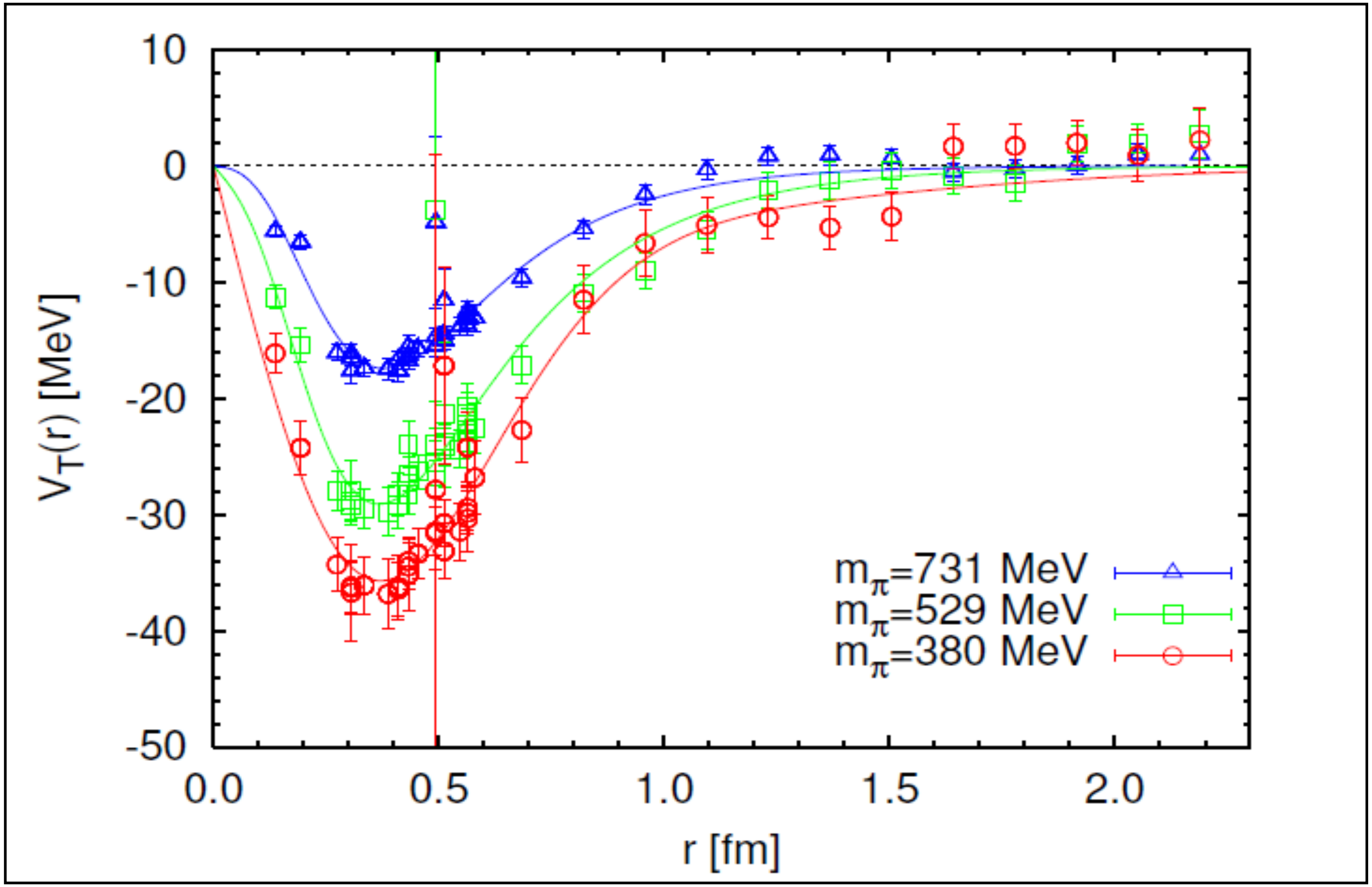


from
R.Machleidt,
Adv.Nucl.Phys.**19**

Fig. 3.7. The contributions from π and ρ (dashed) to the $T=0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Quark mass dependence

Quenched



Fit function

$$V_T(r) = b_1(1-e^{-b_2r^2})^2\left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r} + b_3(1-e^{-b_4r^2})^2\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r},$$

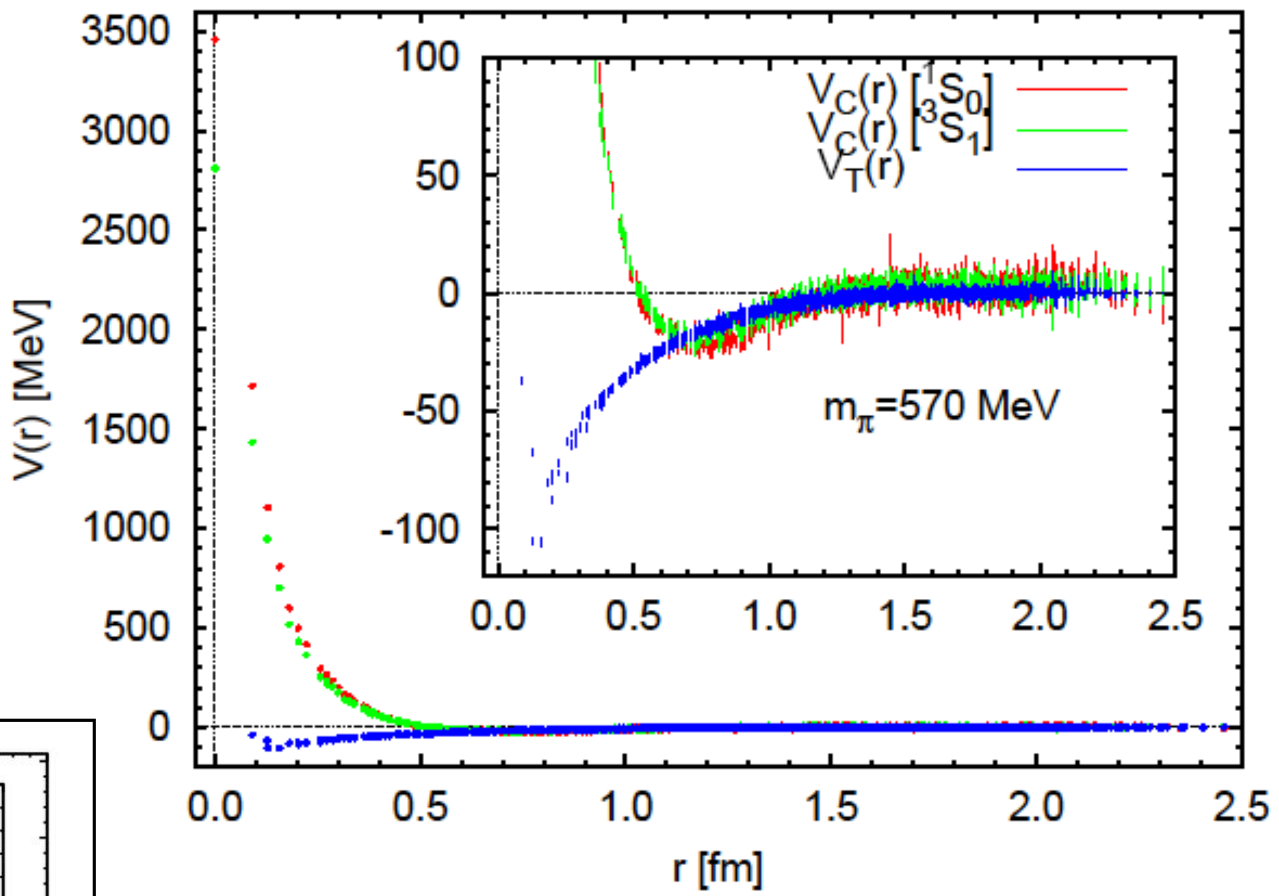
- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange

Full QCD Calculation

Full QCD

$m_\pi = 570\ \mathrm{MeV},\ L = 2.9\ \mathrm{fm}$

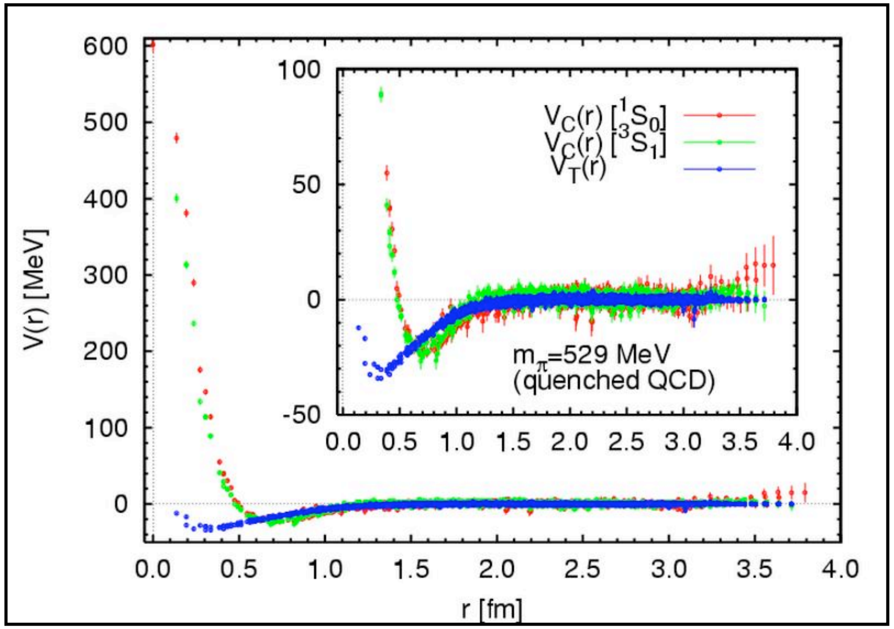
a=0.1fm



Quenched QCD

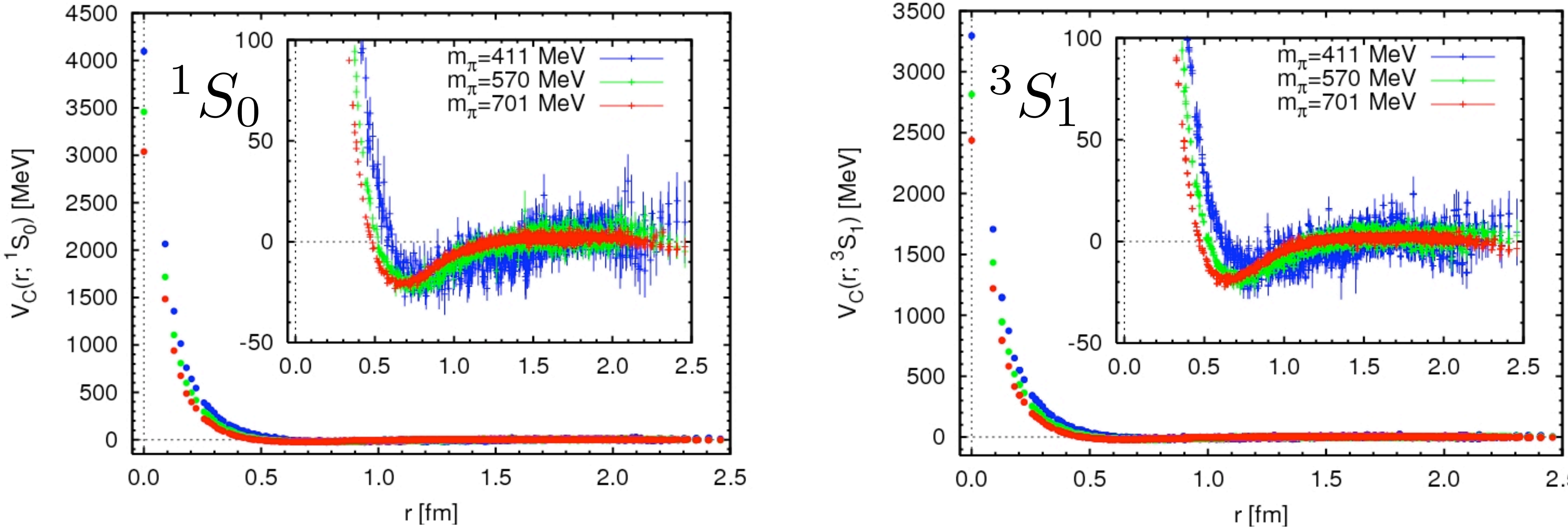
L=4.4fm

$m_\pi \simeq 0.53\ \mathrm{GeV}$ a=0.137 fm

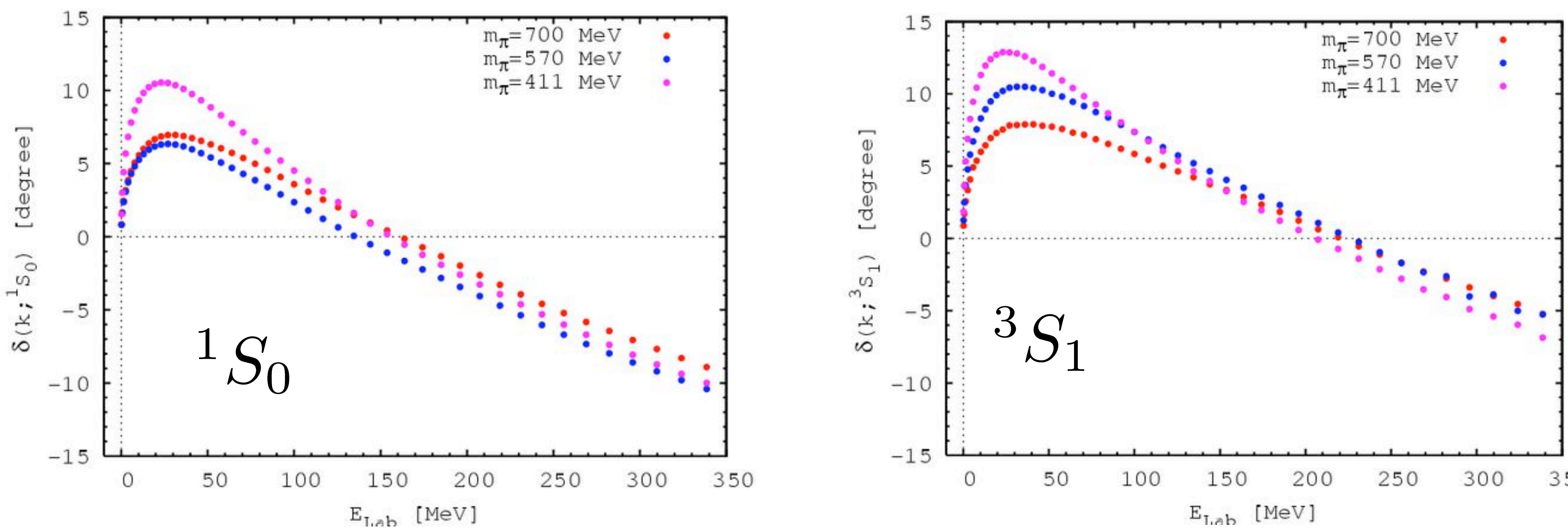


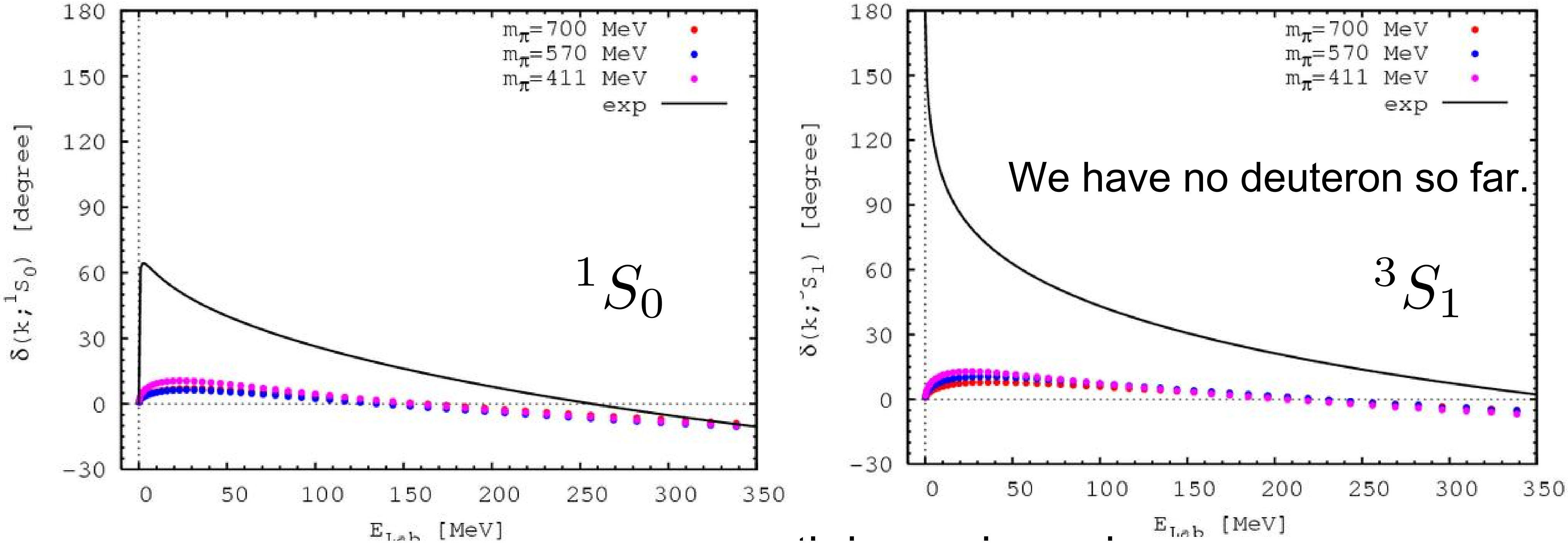
* Large repulsive core than quenched
* Large tensor force than quenched

Phase shift from V(r) in full QCD



a=0.1 fm, L=2.9 fm





They have reasonable shapes. The strength is much weaker.

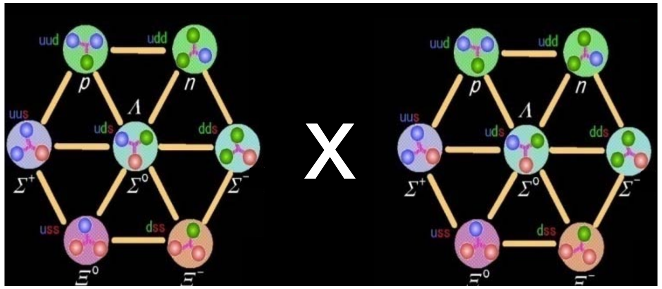
calculation at physical quark mass is important. (future work)

4. Inelastic scattering: octet baryon interactions

Baryon-Baryon interactions in an SU(3) symmetric world

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric   Anti-symmetric

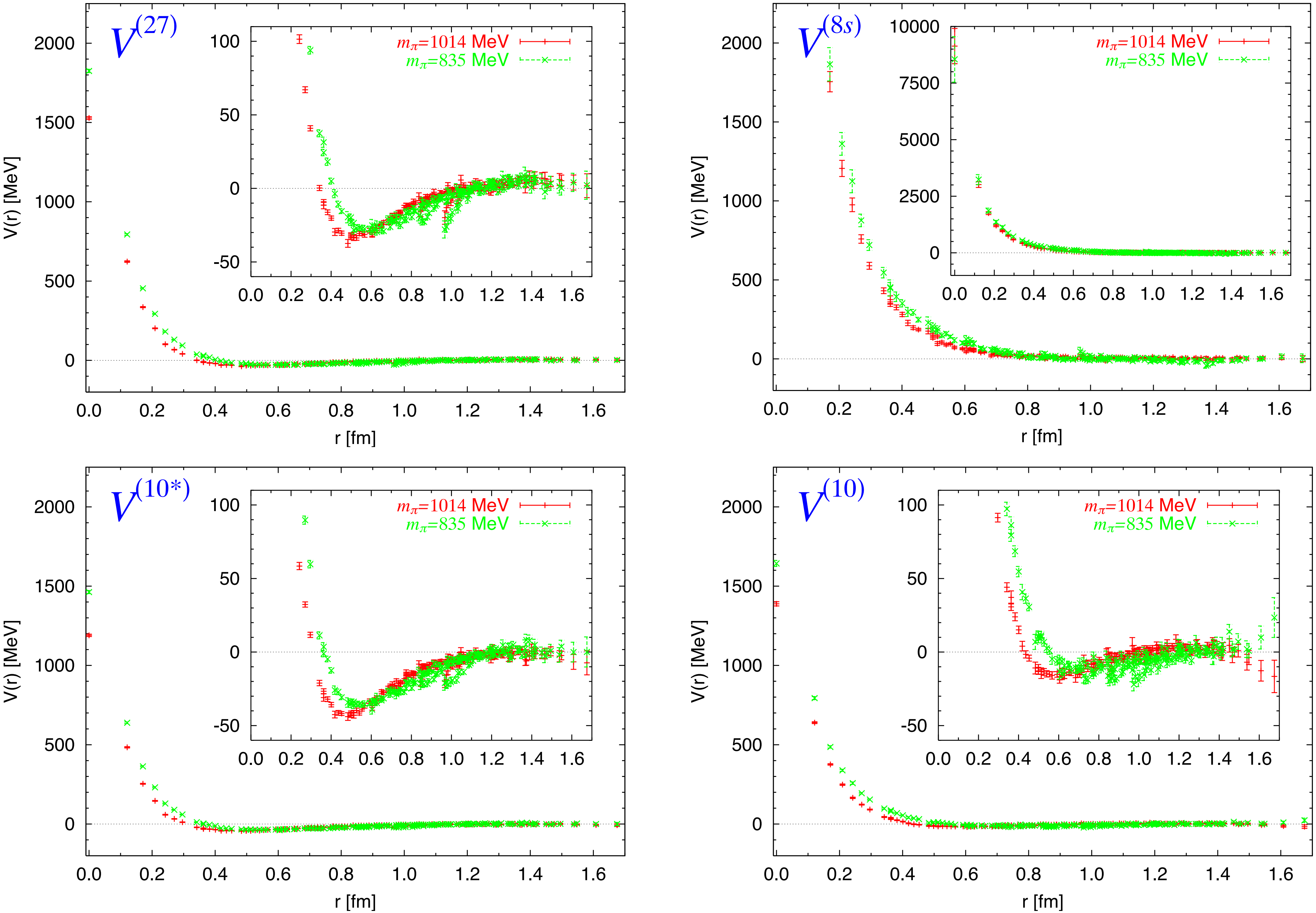
6 independent potential in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

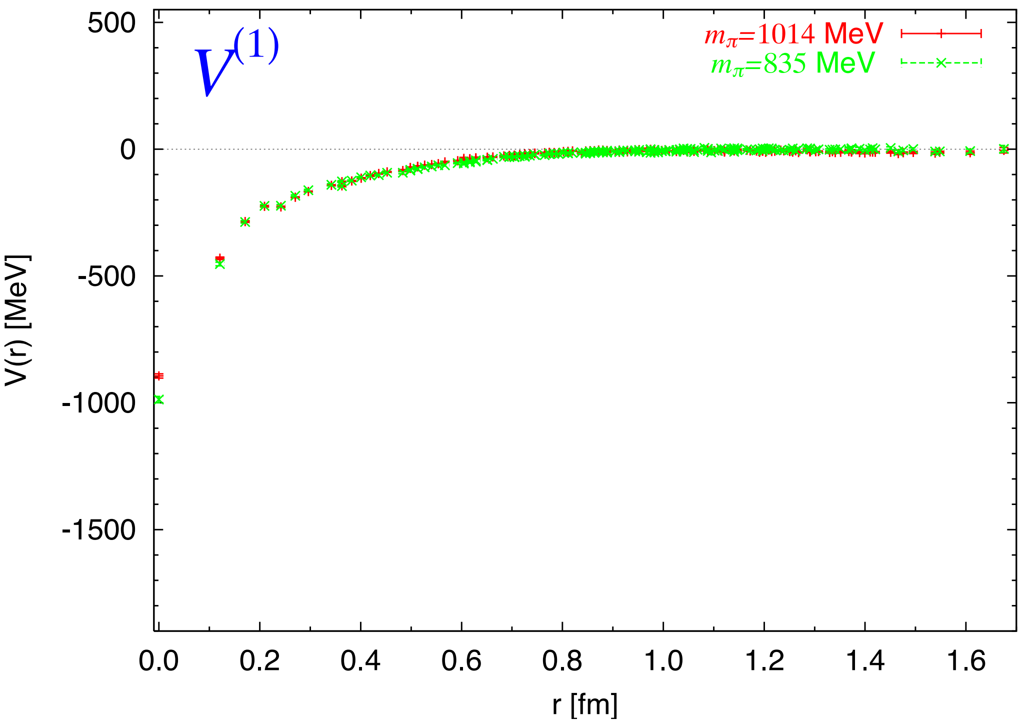
Potentials

Inoue for HAL QCD Collaboration

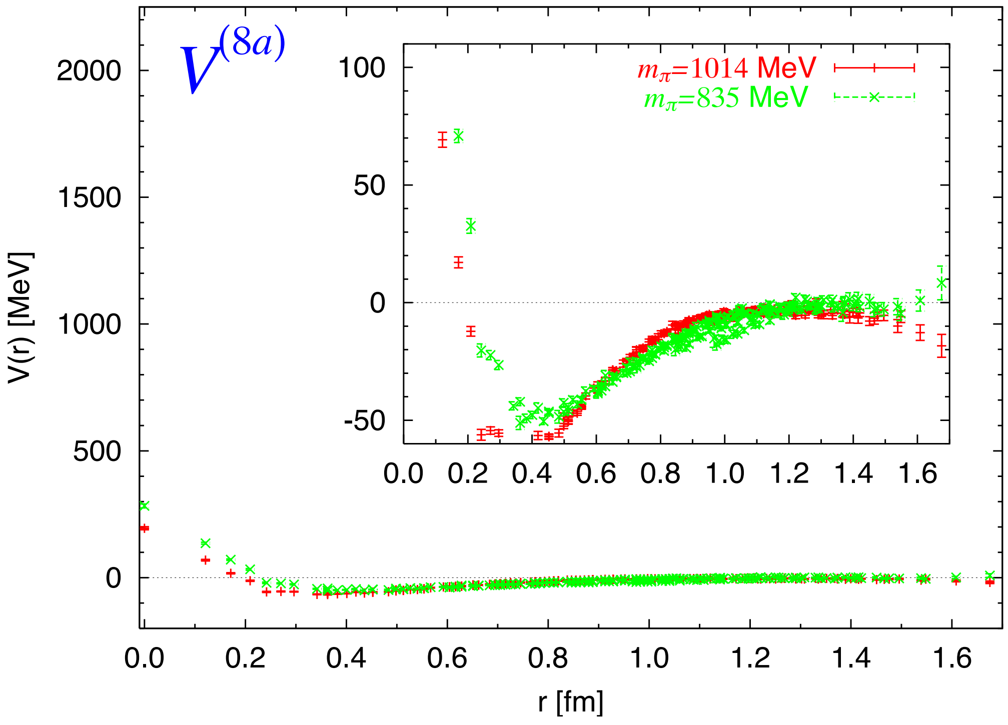


27, 10*: same as before, NN channel

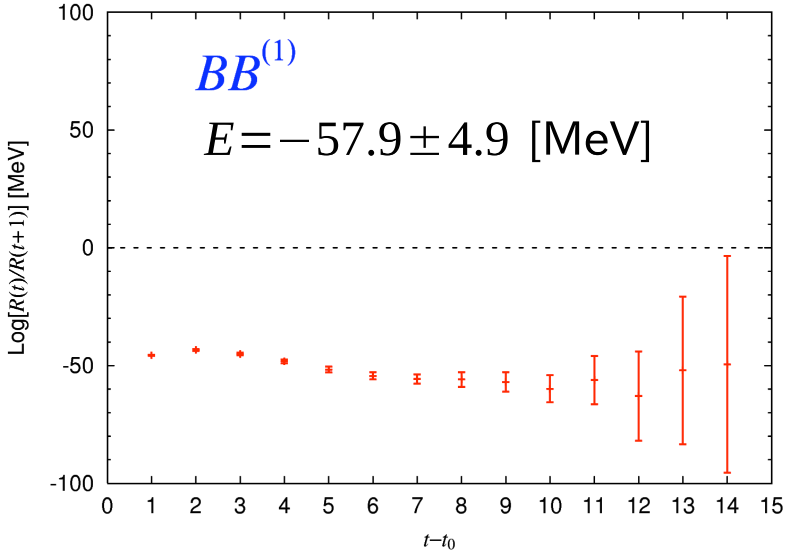
8s, 10: strong repulsive core



1: no repulsive core, attractive core !
No quark mass dependence



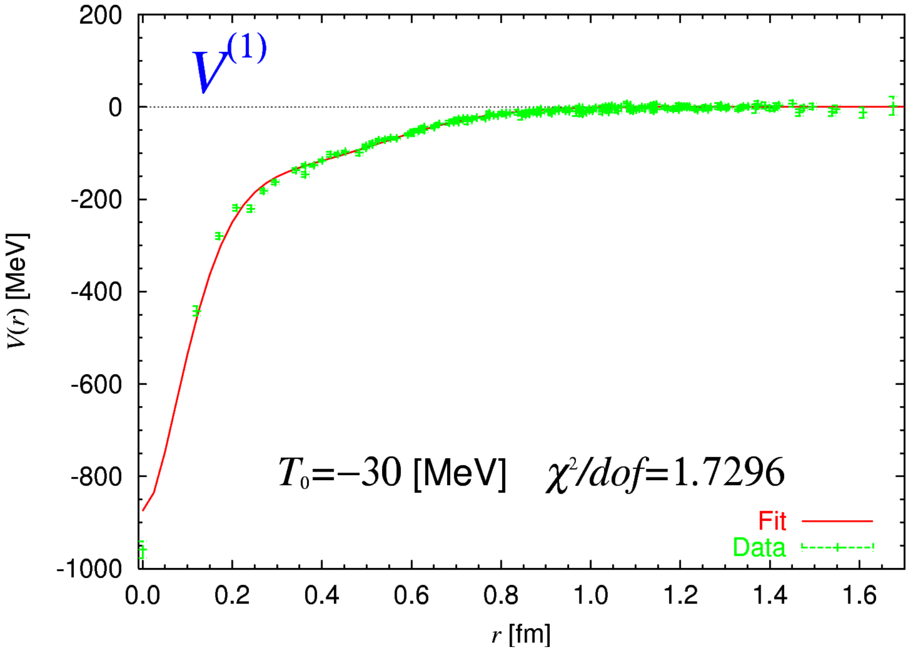
8a: week repulsive core,
deep attractive pocket



Bound state in 1(singlet) channel ?
H-dibaryon ?

However, it is difficult to determine E precisely, due to contaminations from excited states.

Singlet potential with a certain value of E



$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Schroedinger eq. predicts a bound state at E < -30 MeV

E [MeV]	E_0 [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]
E = −30	−0.018	24.7
E = −35	−0.72	4.1
E = −40	−2.49	2.3

Finite size effect is very large on this volume.
(consistent with previous results.)

larger volume calculations are in progress.

Proposal for S=-2 In-elastic scattering

$$m_N = 939 \text{ MeV}, m_\Lambda = 1116 \text{ MeV}, m_\Sigma = 1193 \text{ MeV}, m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

The eigen-state of QCD in the finite box is a mixture of them:

$$|S = -2, I = 0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$

$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$

$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal — off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

off-diagonal — diagonal

μ: reduced mass

$$\begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

$$E_\alpha = \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}, \ \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}$$

$X \neq Y \qquad X, Y = \Lambda\Lambda \text{ or } \Xi N$

$$\alpha = 1, 2$$

Using the potentials:

$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation in the infinite volume with an appropriate boundary condition.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several "in"-states.

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

Lattice is a tool to extract the interaction kernel ("T-matrix" or "potential").

Preliminary results from HAL QCD Collaboration

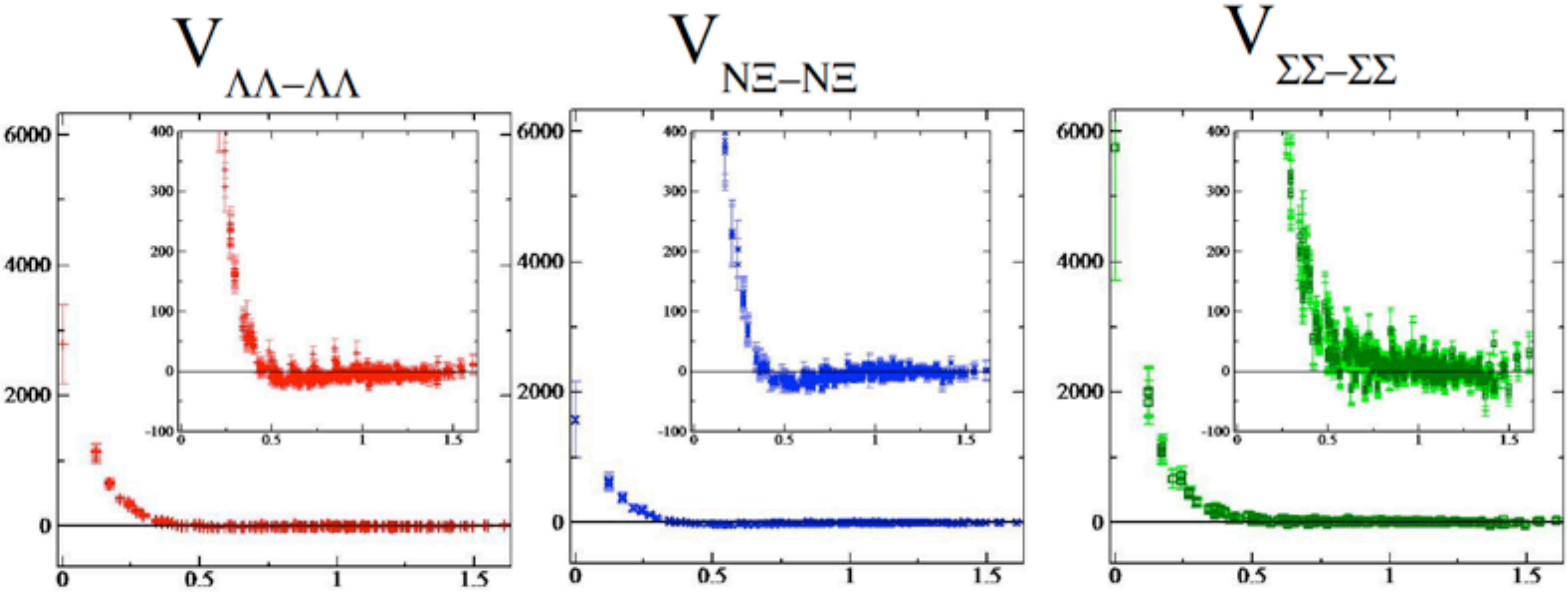
Sasaki for HAL QCD Collaboration

2+1 flavor full QCD

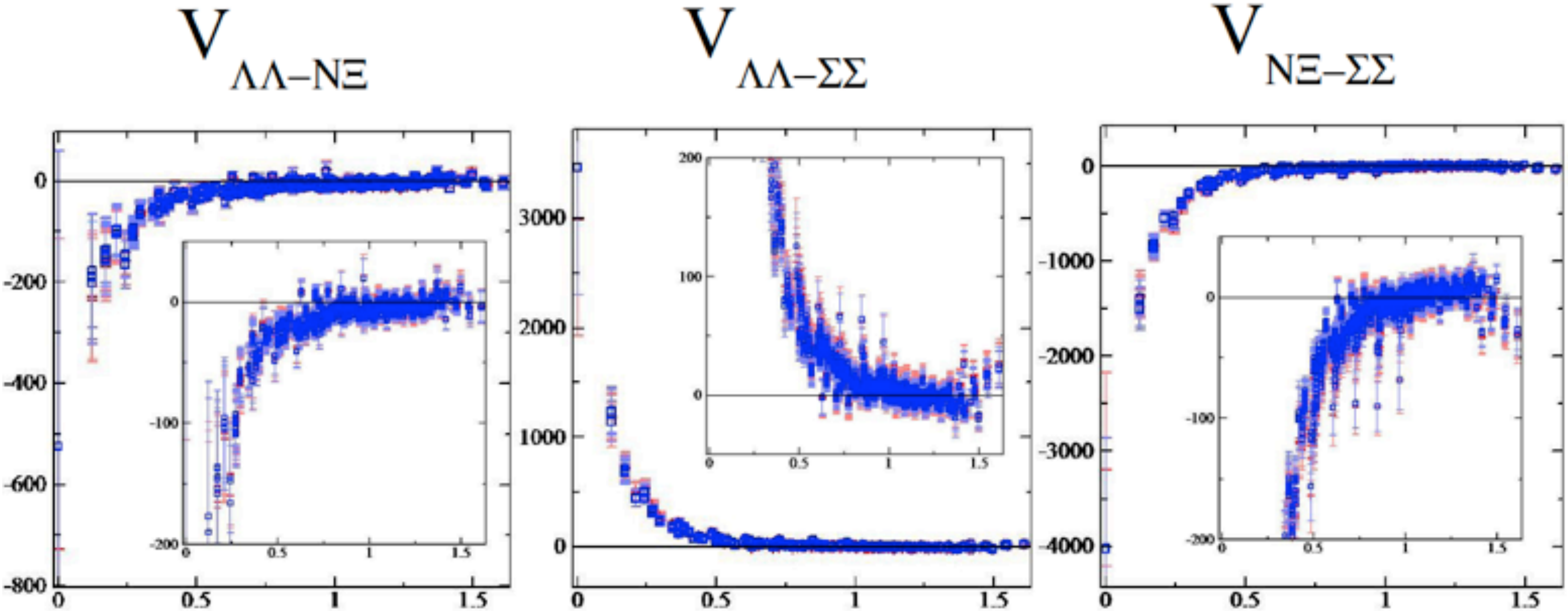
a=0.1 fm, L=2.9 fm

$m_\pi \simeq 870\ \mathrm{MeV}$

Diagonal part of potential matrix



Non-diagonal part of potential matrix



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity ! (non-trivial check)

3-3. H-dibaryon

1. S=-2 singlet state may become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance may appear above ΛΛ but below ΞN threshold.

3. Trial demonstration:

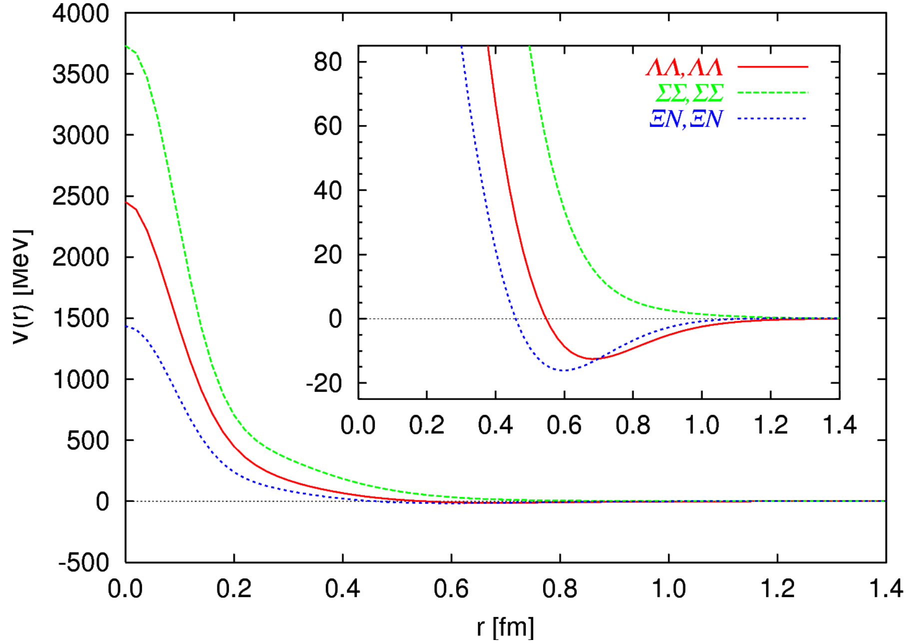
   3.1. Use potential in SU(3) limit

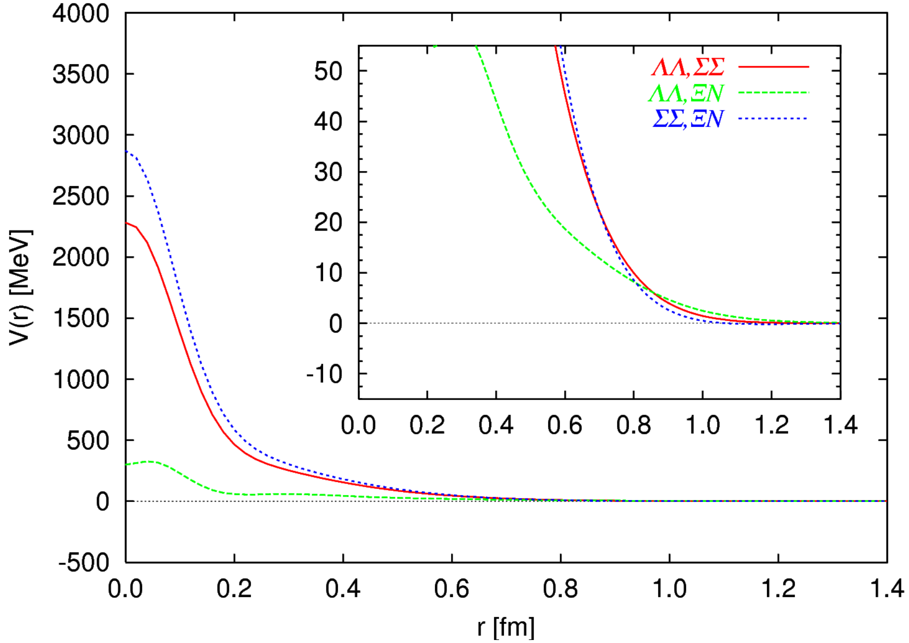
   3.2. Introduce only mass difference from 2+1 simulation

Inoue for HAL QCD Collaboration

Potentials in particle basis in SU(3) limit

$$\begin{pmatrix}\Lambda\Lambda\\ \Sigma\Sigma\\ \Xi N\end{pmatrix} = U \begin{pmatrix}|27\rangle\\ |8\rangle\\ |1\rangle\end{pmatrix}, \quad U\begin{pmatrix}V^{(27)}&&\\&V^{(8)}&\\&&V^{(1)}\end{pmatrix}U^t \longrightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N}\\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N}\\ && V^{\Xi N}\end{pmatrix}$$

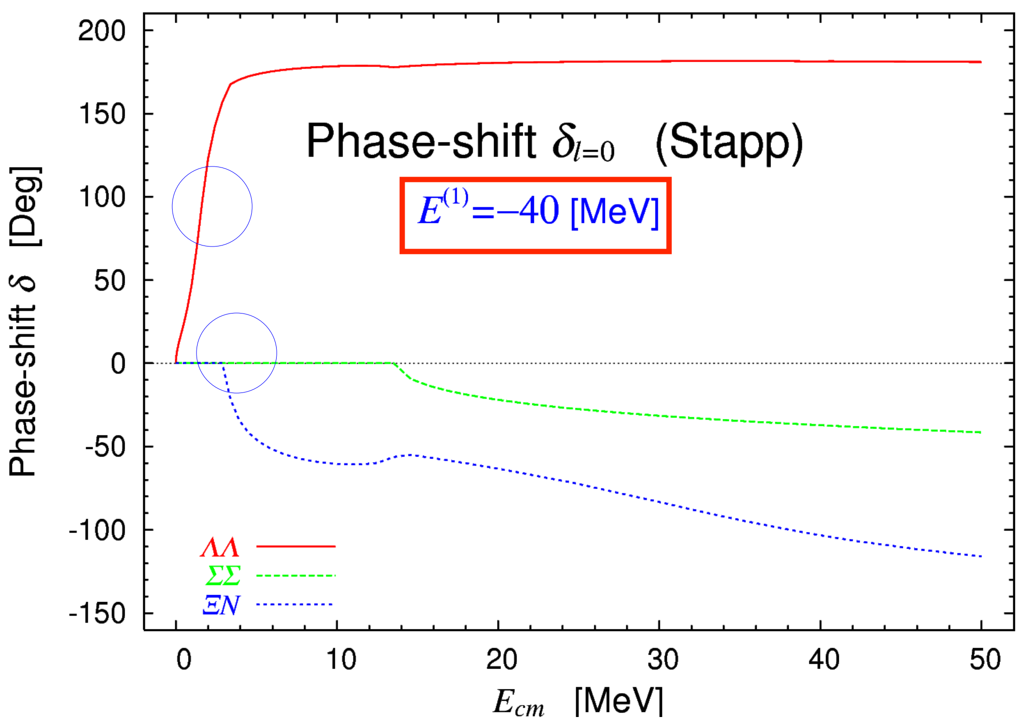


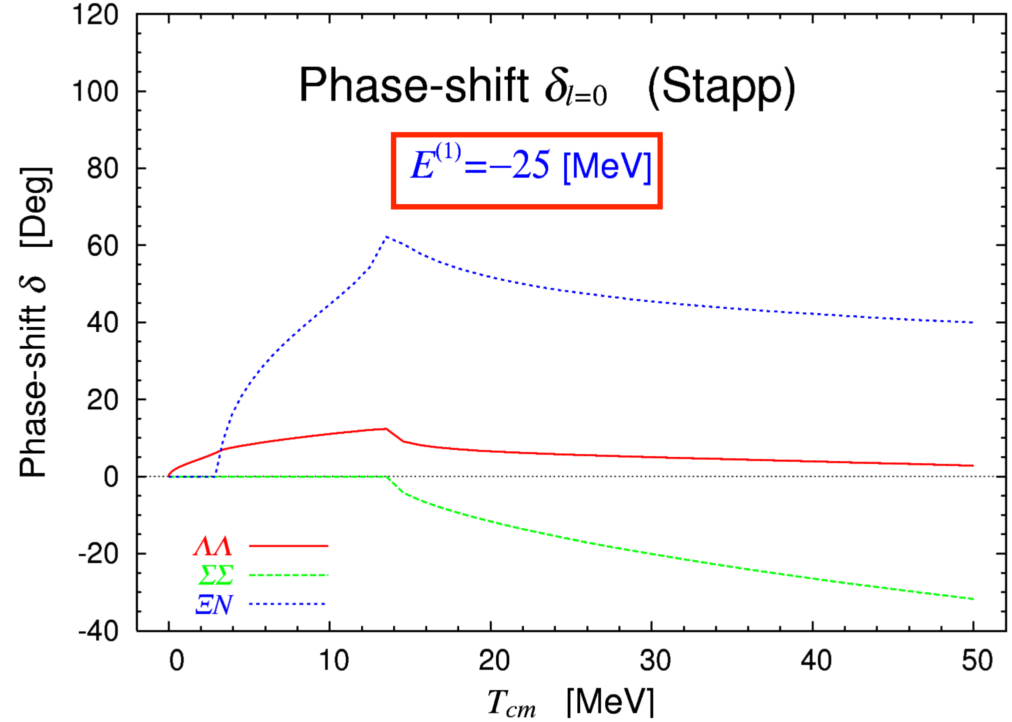


where $T_0^{(1)} = -25$, $T_0^{(8)} = 25$, $T_0^{(27)} = -5$ [MeV] are used

$S = -2, I = 0, {}^1S_0$ scattering

"2+1 flavor"





"2+1 flavor"	resonance	no resonance
	↑	↑
SU(3) limit	bound state	no bound state

5. New method for hadron interactions in lattice QCD

Inelastic scattering II: particle production

$$E \geq E_{th} = 2m_N + m_\pi$$

elastic scattering $NN \leftarrow NN$

NBS wave function

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p}, -\mathbf{p} \leftarrow \mathbf{k}, -\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon} + \mathcal{I}(\mathbf{r})$$

inelastic contribution $NN\pi \leftarrow NN \quad \propto e^{i\mathbf{q}\cdot\mathbf{r}} \qquad |\mathbf{q}| = O(E - E_{th})$

Consider additional NBS wave function

$$\varphi_{E,\pi}(\mathbf{r}, \mathbf{y}) = \langle 0 | N(\mathbf{r} + \mathbf{x}, 0)\pi(\mathbf{y} + \mathbf{x}, 0)N(\mathbf{x}, 0) | 6q, E\rangle$$

Note that

$$|6q, E\rangle = c_1 |NN, E\rangle_{\text{in}} + c_2 |NN\pi, E\rangle_{\text{in}} + \cdots$$

Coupled channel equations

$$(E - H_0)\varphi_E(\mathbf{x}) = \int d^3y\, U_{11}(\mathbf{x};\mathbf{y})\varphi_E(\mathbf{y}) + \int d^3y d^3z\, U_{12}(\mathbf{x};\mathbf{y},\mathbf{z})\varphi_{E,\pi}(\mathbf{y},\mathbf{z})$$

$$(E - H_0)\varphi_{E,\pi}(\mathbf{x},\mathbf{y}) = \int d^3z\, U_{21}(\mathbf{x},\mathbf{y};\mathbf{z})\varphi_E(\mathbf{z}) + \int d^3z d^3w\, U_{22}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{w})\varphi_{E,\pi}(\mathbf{z},\mathbf{w})$$

Velocity expansion at LO, two values of E

$i = 1, 2$

$$(E_i - H_0)\varphi_{E_i}(\mathbf{x}) = V_{11}(\mathbf{x})\varphi_{E_i}(\mathbf{x}) + V_{12}(\mathbf{x},\mathbf{x})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{x})$$

$$(E_i - H_0)\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y}) = V_{21}(\mathbf{x},\mathbf{y})\varphi_{E_i}(\mathbf{x}) + V_{22}(\mathbf{x},\mathbf{y})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y})$$

$V_{11}(\mathbf{x}) : NN \leftarrow NN$ $\quad$ $V_{12}(\mathbf{x},\mathbf{x}) : NN \leftarrow NN\pi$

$V_{21}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN$ $\quad$ $V_{22}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN\pi$

Solve Schroedinger equation with these potentials and a specific B.C.

General prescription

- Consider a QCD eiegnstate with given quantum numbers Q and energy E.
- Take all possible combinations with Q of stable particles whose threshold is below or near E.

$$\text{ex. } Q = 6q: \ \ NN, NN\pi, NN\pi\pi, NNK^+K^-, NN\bar{N}N, \cdots$$

- Calculate NBS wave functions for all combinations.
- Extract coupled-channel potentials in a finite volume.
- Solve Schroedinger equation with these potentials in the infinite volume with a suitable B.C. to obtain physical observables.

In practice, of course, final states more than 2 particles are very difficult to deal with.

6. Theoretical understanding of repulsive core

Summary

- Potentials from NBS wave function are useful tools to extract hadron interactions in lattice QCD. Finite size effect is smaller and quark mass dependence is milder than the phase shift.
  - Combined with Schroedinger equation in the infinite box. Rotational symmetry is recovered.

- Inelastic scattering can also be analysed in terms of coupled channel “potentials”.

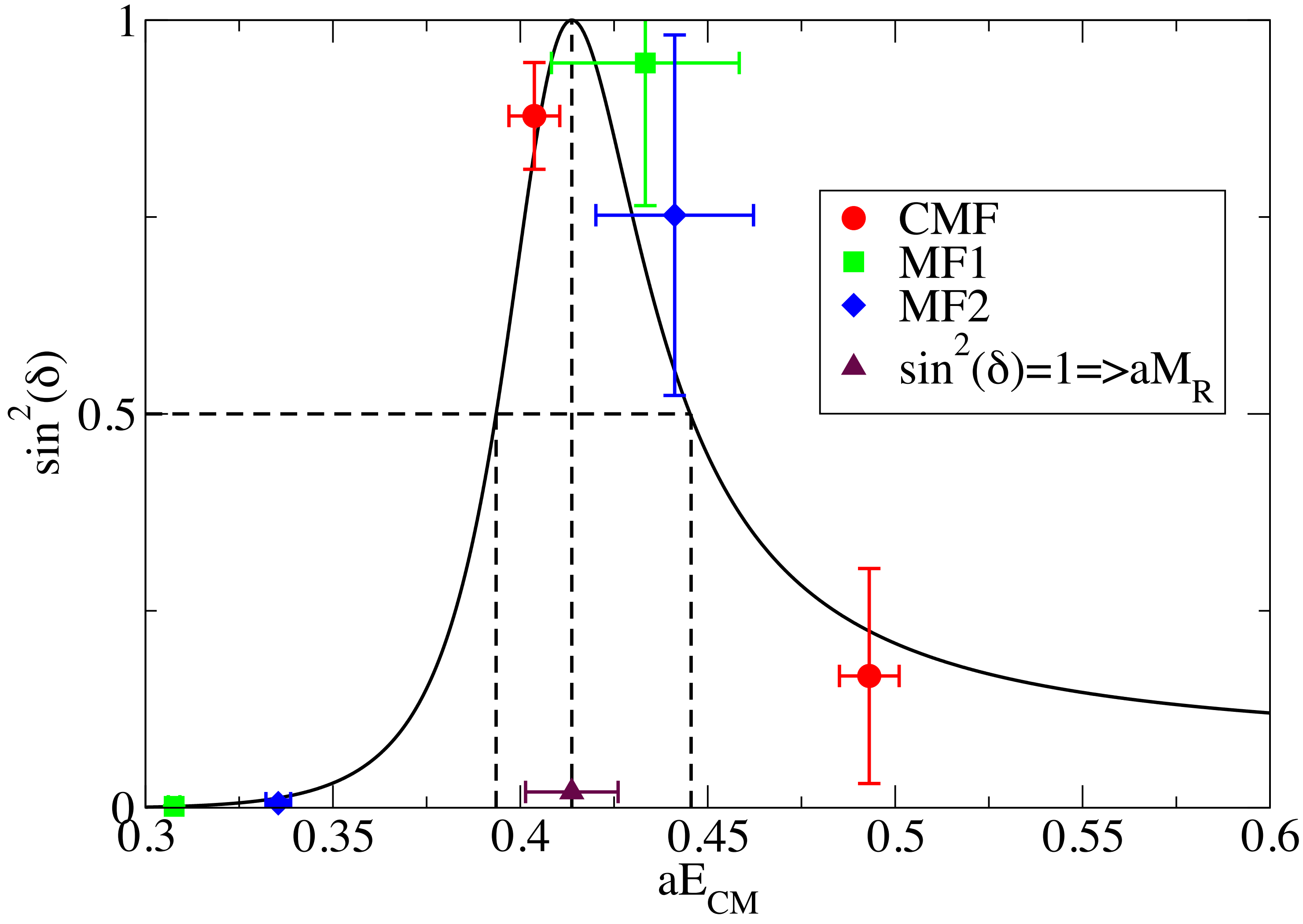
  - ΛΛ scattering, H-dibaryon as a resonance

- unstabel particle as a resonace
  - ρ meson, Δ, Roper etc.
  - exotic: penta-quark, X, Y etc.
- 3-Baryon forces : NNN (Doi) , BBB-> Neutron star
- Weak decay ?

$\pi^+\pi^-$ scattering (ρ meson width)

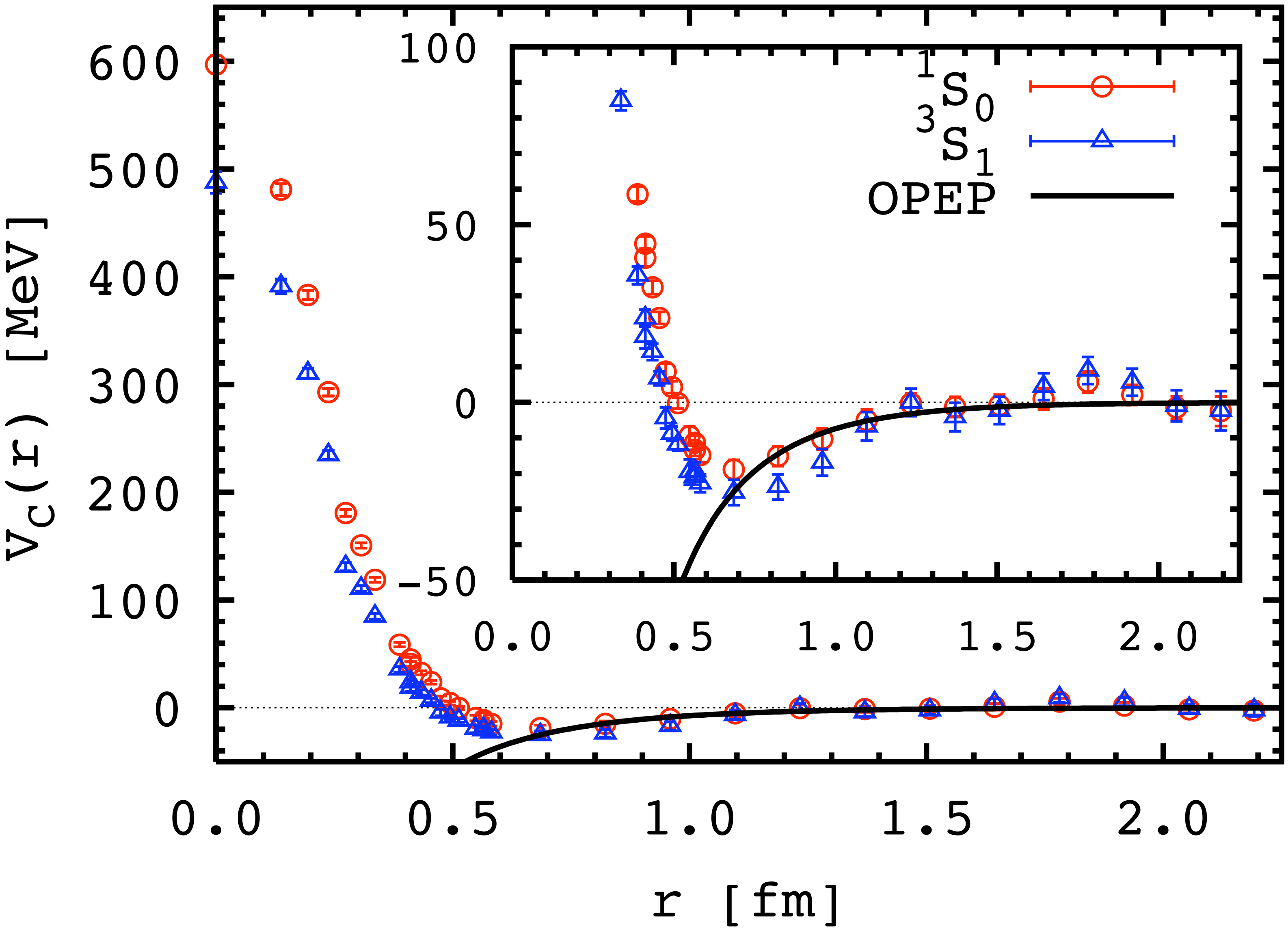
Finite volume method

ETMC: Feng-Jansen-Renner, PLB684(2010)



$$\varphi_E(\mathbf{x}) = \langle 0|\pi(\mathbf{x},0)\pi(\mathbf{0},0)|\rho, E\rangle \Rightarrow V(\mathbf{x}) \Rightarrow \sin^2\delta(s)?$$

QCD meets Nuclei !

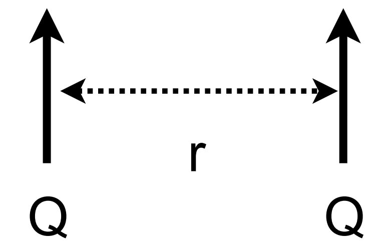


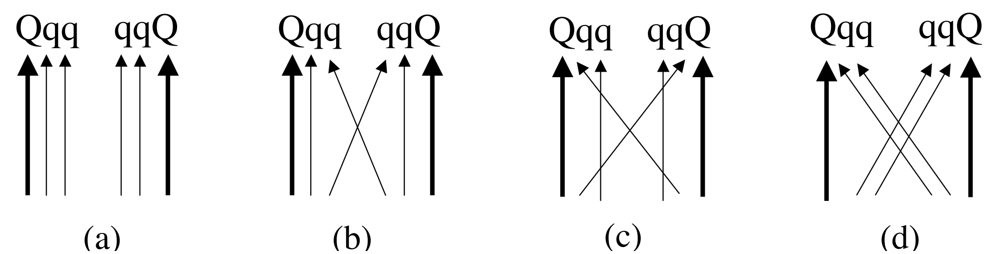
"The achievement is both a computational *tour de force* and a triumph for theory." (Nature Research Higlight 2007)

Definition of "Potential" in (lattice) QCD ?

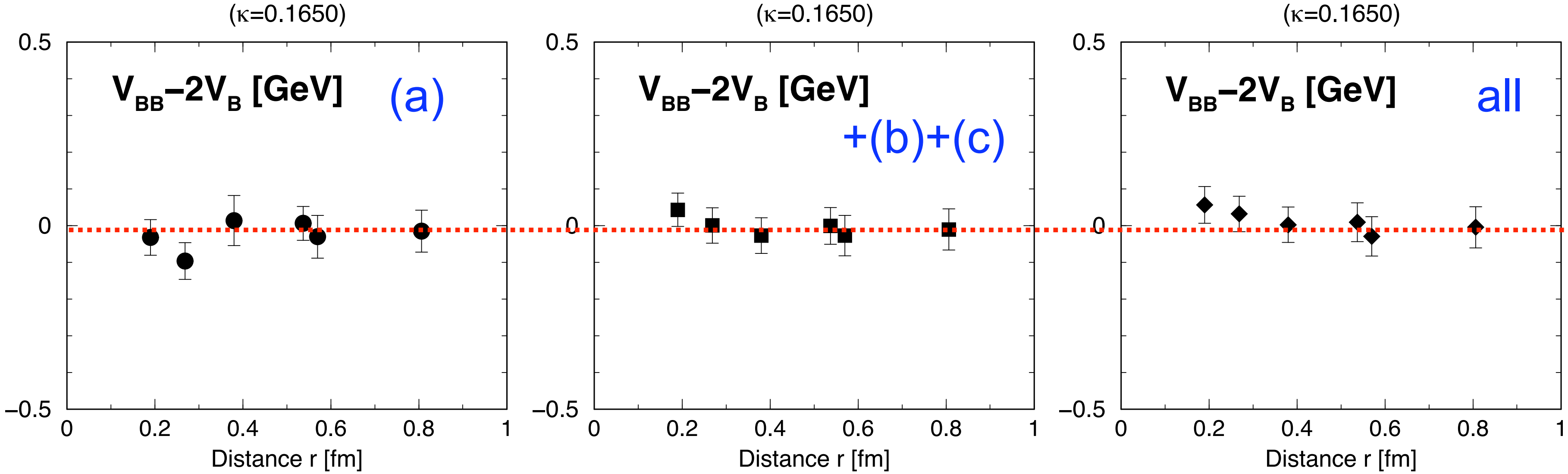
Previous attempt Takahashi-Doi-Suganuma, AIP Conf.Proc. 842,249(2006)

calculate energy of Qqq +Qqq as a function of r between 2Q.
Q:static quark, q: light quark





Quenched result



Almost no dependence on r !

cf. Recent successful result in the strong coupling limit
(deForcrand-Fromm, PRL104(2010)112005)